Equation of a Tangent to a Circle

Practice Worksheet (Problem Solving)

I Do

The circle has centre $(3, 2)$ and radius 5.
The point A has coordinates $(7, 5)$.

Find the equation of the tangent to the circle at A.
Give your answer in the form $ax + by = c$, where a, b, and c are integers.

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(Total for Question is 4 marks)

Video walkthrough

We Do

The circle has centre $(1, -2)$ and radius 10.
The point P has coordinates $(7, 6)$.

Find the equation of the tangent to the circle at P.
Give your answer in the form $ax + by = c$, where a, b, and c are integers.

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(Total for Question is 3 marks)

You Do 1

The circle has centre $(0, 0)$ and radius 13.
The point Q has coordinates $(-5, 12)$.

Find the equation of the tangent to the circle at Q.
Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

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(Total for Question is 3 marks)

You Do 2

The circle has centre $(-4, 3)$ and radius 5.
The point R has coordinates $(-1, -1)$.

Find the equation of the tangent to the circle at R.
Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

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(Total for Question is 3 marks)

You Do 3

The circle has centre $(2, -1)$ and radius $\sqrt{20}$.
The point S has coordinates $(4, 3)$.

Find the equation of the tangent to the circle at S.
Give your answer in the form $ax + by = c$, where a, b, and c are integers.

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(Total for Question is 3 marks)

Challenge 1

A circle has the equation $x^2 + y^2 - 6x + 8y = 0$.
The point T has coordinates $(6, 0)$.

Find the equation of the tangent to the circle at T.
Give your answer in the form $ax + by + c = 0$.

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(Total for Question is 5 marks)

Extreme Challenge

The circle C_1 has equation $x^2 + y^2 = 25$.
The line L has equation $3x - 4y = 7$.

There are two tangents to the circle C_1 which are parallel to the line L.

Find the equations of these two tangents.
Give your answers in the form $ax + by + c = 0$, where a, b, and c are integers.
You must show all your algebraic steps clearly.

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(Total for Question is 6 marks)

Worked Answers

I Do

Find the gradient of the radius to A:
$m_{rad} = \frac{5-2}{7-3} = \frac{3}{4}$.
The tangent is perpendicular to the radius, so its gradient is the negative reciprocal:
$m_{tan} = -\frac{4}{3}$.
Use $y - y_1 = m(x - x_1)$ with point A$(7, 5)$:
$y - 5 = -\frac{4}{3}(x - 7)$
Multiply by 3: $3(y - 5) = -4(x - 7) \implies 3y - 15 = -4x + 28$.
Rearrange to $ax + by = c$: $\mathbf{4x + 3y = 43}$.

We Do

Gradient of radius $= \frac{6-(-2)}{7-1} = \frac{8}{6} = \frac{4}{3}$.
Gradient of tangent $= -\frac{3}{4}$.
Equation: $y - 6 = -\frac{3}{4}(x - 7)$
$4(y - 6) = -3(x - 7) \implies 4y - 24 = -3x + 21$.
Rearrange: $\mathbf{3x + 4y = 45}$.

You Do 1

Gradient of radius $= \frac{12-0}{-5-0} = -\frac{12}{5}$.
Gradient of tangent $= \frac{5}{12}$.
Equation: $y - 12 = \frac{5}{12}(x - (-5))$
$12(y - 12) = 5(x + 5) \implies 12y - 144 = 5x + 25$.
Rearrange to $ax + by + c = 0$: $\mathbf{5x - 12y + 169 = 0}$.

You Do 2

Gradient of radius $= \frac{-1-3}{-1-(-4)} = \frac{-4}{3}$.
Gradient of tangent $= \frac{3}{4}$.
Equation: $y-(-1) = \frac{3}{4}(x-(-1))$
$4(y+1) = 3(x+1) \implies 4y+4 = 3x+3$.
Rearrange: $\mathbf{3x-4y-1=0}$.

You Do 3

Gradient of radius $= \frac{3-(-1)}{4-2} = \frac{4}{2} = 2$.
Gradient of tangent $= -\frac{1}{2}$.
Equation: $y-3 = -\frac{1}{2}(x-4)$
$2(y-3) = -1(x-4) \implies 2y-6 = -x+4$.
Rearrange: $\mathbf{x+2y=10}$.

Challenge 1

Complete the square for $x^2-6x+y^2+8y=0$:
$(x-3)^2-9+(y+4)^2-16=0$
$(x-3)^2+(y+4)^2=25$.
Centre is (3, -4).
Gradient of radius to $T(6,0) = \frac{0-(-4)}{6-3} = \frac{4}{3}$.
Gradient of tangent $= -\frac{3}{4}$.
Equation: $y-0 = -\frac{3}{4}(x-6) \implies 4y = -3(x-6) \implies 4y = -3x+18$.
Rearrange: $\mathbf{3x+4y-18=0}$.

Extreme Challenge

1. Find the gradient of line L:

$3x - 4y = 7 \implies 4y = 3x - 7 \implies y = \frac{3}{4}x - \frac{7}{4}$.

Gradient $= \frac{3}{4}$.

The tangents must also have a gradient of $\frac{3}{4}$.

2. Find the points of contact:

The radius to the point of contact must be perpendicular to the tangent.

Gradient of radius $= -\frac{4}{3}$.

Since the circle is $x^2 + y^2 = 25$, the centre is $(0, 0)$.

For a point (x, y) on the circle, the gradient from the origin is $\frac{y}{x} = -\frac{4}{3} \implies y = -\frac{4}{3}x$.

Substitute this into the circle equation:

$x^2 + \left(-\frac{4}{3}x\right)^2 = 25 \implies x^2 + \frac{16}{9}x^2 = 25 \implies \frac{25}{9}x^2 = 25 \implies x^2 = 9$.

$x = \pm 3$.

If $x = 3, y = -\frac{4}{3}(3) = -4$. Point 1: $(3, -4)$.

If $x = -3, y = -\frac{4}{3}(-3) = 4$. Point 2: $(-3, 4)$.

3. Find equations of the tangents:

Tangent 1 at $(3, -4)$ with gradient $\frac{3}{4}$:

$y - (-4) = \frac{3}{4}(x - 3) \implies 4(y + 4) = 3(x - 3) \implies 4y + 16 = 3x - 9$.

$\mathbf{3x - 4y - 25 = 0}$.

Tangent 2 at $(-3, 4)$ with gradient $\frac{3}{4}$:

$y - 4 = \frac{3}{4}(x - (-3)) \implies 4(y - 4) = 3(x + 3) \implies 4y - 16 = 3x + 9$.

$\mathbf{3x - 4y + 25 = 0}$.