Exact Trigonometric Values

Practice Worksheet (Problem Solving)

I Do

Find the exact value of

$$\frac{\sin 30^\circ + \cos 60^\circ}{\tan 60^\circ}$$

Give your answer in its simplest form with a rational denominator.

.....

(Total for Question is 3 marks)

We Do

Find the exact value of

$$\frac{\sin 30^\circ - \tan 45^\circ}{\cos 30^\circ}$$

Give your answer in its simplest form with a rational denominator.

.....

(Total for Question is 3 marks)

You Do 1

Find the exact value of

$$\frac{\tan 45^\circ + \sin 30^\circ}{\cos 30^\circ}$$

Give your answer in its simplest form with a rational denominator.

.....

(Total for Question is 3 marks)

You Do 2

Find the exact value of

$$\frac{\sin 45^\circ + \cos 45^\circ}{\tan 60^\circ}$$

Give your answer in its simplest form with a rational denominator.

.....

(Total for Question is 3 marks)

You Do 3

Find the exact value of

$$\frac{\tan^2 30^\circ + \cos 60^\circ}{\sin 60^\circ}$$

Give your answer in its simplest form with a rational denominator.

.....

(Total for Question is 3 marks)

Challenge 1

Find the exact value of

$$\frac{\tan 60^\circ}{1 - \cos 30^\circ}$$

Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.
Show clear algebraic working.

.....

(Total for Question is 4 marks)

Challenge 2

Solve the equation for x:

$$x \cos 30^\circ = x \sin 30^\circ + \tan 60^\circ$$

Give your answer in the form $c + \sqrt{d}$, where c and d are integers.
Show clear algebraic working.

$x =$ .....

(Total for Question is 5 marks)

Extreme Challenge

Show that the exact value of

$$\frac{\cos 30^\circ}{\tan 60^\circ - \sin 45^\circ}$$

can be written in the form $\frac{m+\sqrt{n}}{10}$, where m and n are integers.

You must show all your algebraic steps clearly.

.....

(Total for Question is 5 marks)

Worked Answers

I Do

Substitute the exact trigonometric values:
$\sin 30° = \frac{1}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$.
The expression becomes:
$\frac{\frac{1}{2}+\frac{1}{2}}{\sqrt{3}}$
Simplify the numerator: $\frac{1}{2} + \frac{1}{2} = 1$.
The expression is now $\frac{1}{\sqrt{3}}$.
To rationalise the denominator, multiply the numerator and denominator by $\sqrt{3}$:
$\frac{1\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{\sqrt{\mathbf{3}}}{\mathbf{3}}$.

We Do

Substitute the exact values:
$\sin 30° = \frac{1}{2}$, $\tan 45° = 1$, $\cos 30° = \frac{\sqrt{3}}{2}$.
The expression becomes:
$\frac{\frac{1}{2}-1}{\frac{\sqrt{3}}{2}} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$
When dividing fractions, multiply by the reciprocal of the denominator:
$-\frac{1}{2} \times \frac{2}{\sqrt{3}} = -\frac{2}{2\sqrt{3}} = -\frac{1}{\sqrt{3}}$.
Rationalise the denominator by multiplying top and bottom by $\sqrt{3}$:
$-\frac{\sqrt{\mathbf{3}}}{\mathbf{3}}$.

You Do 1

Substitute the exact values:
$\tan 45° = 1$, $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$.
The expression becomes:
$\frac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}}$
Multiply by the reciprocal:
$\frac{3}{2} \times \frac{2}{\sqrt{3}} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}}$.
Rationalise the denominator:
$\frac{3\sqrt{3}}{3} = \mathbf{\sqrt{3}}$.

You Do 2

Substitute the exact values:
$\sin 45° = \frac{\sqrt{2}}{2}$ (or $\frac{1}{\sqrt{2}}$), $\cos 45° = \frac{\sqrt{2}}{2}$, $\tan 60° = \sqrt{3}$.
The expression becomes:
$\frac{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$
Rationalise the denominator by multiplying top and bottom by $\sqrt{3}$:
$\frac{\sqrt{2}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{\sqrt{\mathbf{6}}}{\mathbf{3}}$.

You Do 3

Substitute the exact values:
$\tan 30° = \frac{1}{\sqrt{3}}$, $\cos 60° = \frac{1}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$.
First, calculate $\tan^2 30° = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$.
The expression becomes:
$\frac{\frac{1}{3}+\frac{1}{2}}{\frac{\sqrt{3}}{2}}$
Find a common denominator for the numerator (which is 6):
$\frac{\frac{2}{6}+\frac{3}{6}}{\frac{\sqrt{3}}{2}} = \frac{\frac{5}{6}}{\frac{\sqrt{3}}{2}}$
Multiply by the reciprocal:
$\frac{5}{6} \times \frac{2}{\sqrt{3}} = \frac{10}{6\sqrt{3}} = \frac{5}{3\sqrt{3}}$.
Rationalise the denominator by multiplying top and bottom by $\sqrt{3}$:
$\frac{5\sqrt{3}}{3\times 3} = \frac{\mathbf{5\sqrt{3}}}{\mathbf{9}}$.

Challenge 1

Substitute the exact values:
$\frac{\sqrt{3}}{1-\frac{\sqrt{3}}{2}}$
Multiply the numerator and denominator by 2 to clear the fraction inside the denominator:
$\frac{2\sqrt{3}}{2-\sqrt{3}}$
Rationalise the denominator by multiplying by the conjugate $(2+\sqrt{3})$:
$\frac{2\sqrt{3}(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$
Expand the denominator (Difference of Two Squares): $2^2 - (\sqrt{3})^2 = 4 - 3 = 1$.
Expand the numerator: $2\sqrt{3}(2) + 2\sqrt{3}(\sqrt{3}) = 4\sqrt{3} + 2(3) = 4\sqrt{3} + 6$.
Final answer: $\mathbf{6 + 4\sqrt{3}}$.

Challenge 2

Substitute the exact values into the equation:
$x\left(\frac{\sqrt{3}}{2}\right) = x\left(\frac{1}{2}\right) + \sqrt{3}$
Multiply the entire equation by 2 to clear fractions:
$x\sqrt{3} = x + 2\sqrt{3}$
Rearrange to get all x terms on one side:
$x\sqrt{3} - x = 2\sqrt{3}$
Factorise x out on the left hand side:
$x(\sqrt{3} - 1) = 2\sqrt{3}$
Divide to solve for x:
$x = \frac{2\sqrt{3}}{\sqrt{3}-1}$
Rationalise the denominator by multiplying by the conjugate $(\sqrt{3} + 1)$:
$x = \frac{2\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$
Expand the denominator: $(\sqrt{3})^2 - 1^2 = 3 - 1 = 2$.
Expand the numerator: $2\sqrt{3}(\sqrt{3}) + 2\sqrt{3}(1) = 6 + 2\sqrt{3}$.
$x = \frac{6+2\sqrt{3}}{2} = \mathbf{3 + \sqrt{3}}$.

Extreme Challenge

Substitute the exact values:
$\frac{\frac{\sqrt{3}}{2}}{\sqrt{3}-\frac{\sqrt{2}}{2}}$
Multiply the numerator and the denominator by 2 to clear the internal fractions:
$\frac{\sqrt{3}}{2\sqrt{3}-\sqrt{2}}$
Rationalise the denominator by multiplying top and bottom by the conjugate $(2\sqrt{3} + \sqrt{2})$:
$\frac{\sqrt{3}(2\sqrt{3}+\sqrt{2})}{(2\sqrt{3}-\sqrt{2})(2\sqrt{3}+\sqrt{2})}$
Expand the denominator (Difference of Two Squares):
$(2\sqrt{3})^2 - (\sqrt{2})^2 = (4 \times 3) - 2 = 12 - 2 = 10$.
Expand the numerator:
$\sqrt{3}(2\sqrt{3}) + \sqrt{3}(\sqrt{2}) = 2(3) + \sqrt{6} = 6 + \sqrt{6}$.
Combine to get the final fraction:
$\mathbf{\frac{6+\sqrt{6}}{10}}$. (Proved)