Loci and Grazing Areas

Practice Worksheet (Grade 9 Mastery)

I Do

A square barn has sides of length $2a$.
Tom tethers a goat to a point at ground level on one outside corner of the barn by a rope of length $4a$.
The barn is surrounded by a large field of grass.

Find an expression, in terms of a and π, for the maximum area of grass that the goat can graze.
Give your answer in its simplest form.

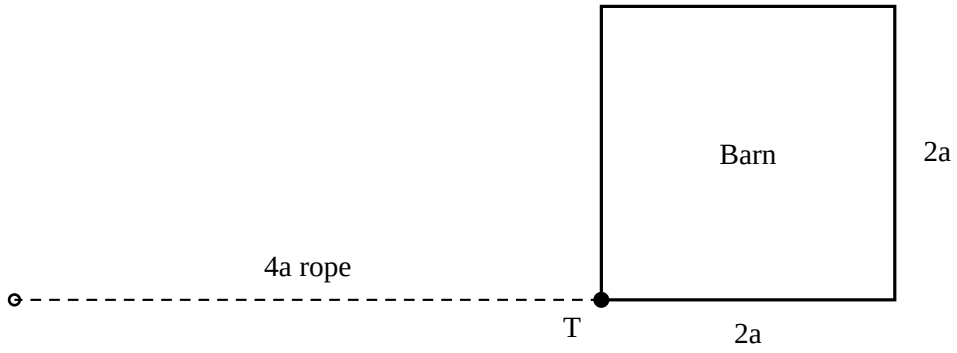


Diagram not to scale. T is the tether point.

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(Total for Question is 5 marks)

We Do

A rectangular barn has a length of $3x$ and a width of $2x$.
Catherine tethers a goat to a point at ground level on one outside corner of the barn by a rope of length $5x$.
The barn is surrounded by a large field of grass.

Find an expression, in terms of x and π, for the maximum area of grass that the goat can graze.
Give your answer in its simplest form.

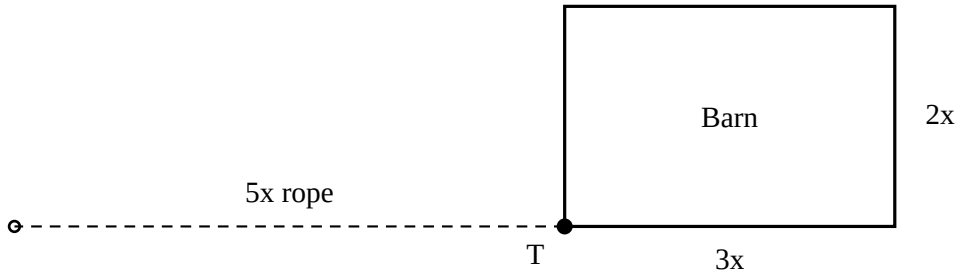


Diagram not to scale. T is the tether point.

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(Total for Question is 5 marks)

You Do 1

A square barn has sides of length $3b$.
John tethers a goat to a point at ground level on one outside corner of the barn by a rope of length $5b$.
The barn is surrounded by a large field of grass.

Find an expression, in terms of b and π, for the maximum area of grass that the goat can graze.
Give your answer as a single fraction in its simplest form.

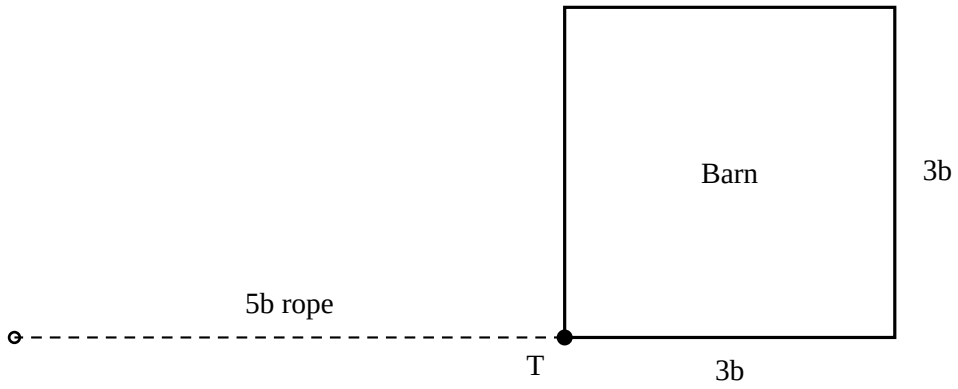


Diagram not to scale. T is the tether point.

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(Total for Question is 4 marks)

You Do 2

A rectangular barn has a length of $4c$ and a width of $2c$.
Ryan tethers a goat to a point at ground level **exactly in the middle** of one of the walls of length $4c$.
The rope has a length of $3c$.
The barn is surrounded by a large field of grass.

Find an expression, in terms of c and π, for the maximum area of grass that the goat can graze.
Give your answer in its simplest form.

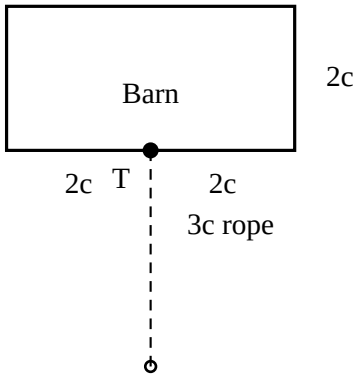


Diagram not to scale. T is the tether point.

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(Total for Question is 5 marks)

You Do 3

A barn is built in the shape of a **regular hexagon** with sides of length y.
Paul tethers a goat to a point at ground level on one outside corner of the barn by a rope of length $2y$.
The barn is surrounded by a large field of grass.

Find an expression, in terms of y and π, for the maximum area of grass that the goat can graze.
Give your answer in its simplest form.

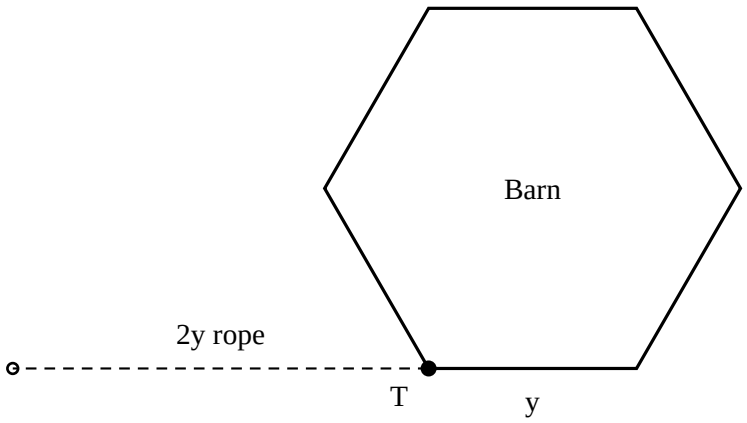


Diagram not to scale. T is the tether point.

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(Total for Question is 5 marks)

Challenge 1

A barn is built in the shape of an **equilateral triangle** with sides of length $2w$.
Sophie tethers a goat to a point at ground level on one outside corner of the barn by a rope of length $4w$.
The barn is surrounded by a large field of grass.

Find an expression, in terms of w and π, for the maximum area of grass that the goat can graze.
Give your answer in its simplest form.

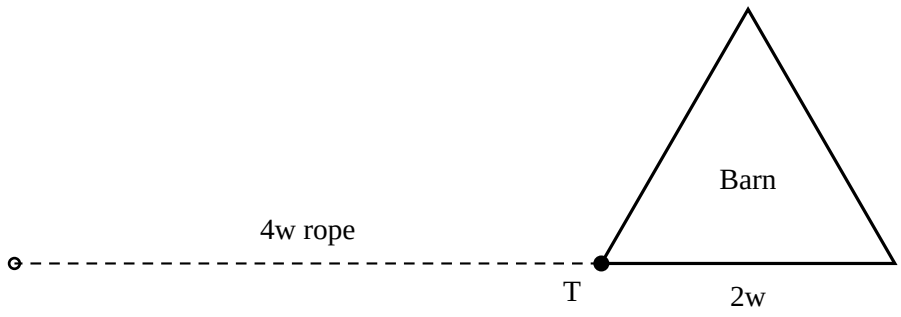


Diagram not to scale. T is the tether point.

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(Total for Question is 5 marks)

Challenge 2

A rectangular barn has a length of $6k$ and a width of $4k$.
Paige tethers a goat to a point at ground level on one of the walls of length $6k$.
The tether point is exactly $2k$ away from the nearest corner.
The rope has a length of $5k$.
The barn is surrounded by a large field of grass.

Find an expression, in terms of k and π, for the maximum area of grass that the goat can graze.
Give your answer in its simplest form.

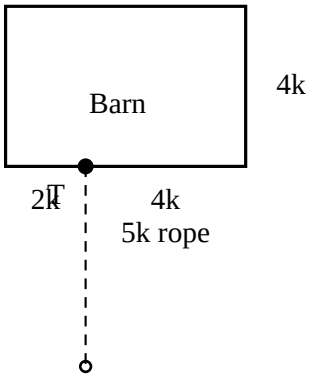


Diagram not to scale. T is the tether point.

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(Total for Question is 5 marks)

Extreme Challenge

A barn is built in the shape of a **regular octagon** with sides of length m.
Katie tethers a goat to a point at ground level on one outside corner of the barn by a rope of length $3m$.
The barn is surrounded by a large field of grass.

Find an expression, in terms of m and π, for the maximum area of grass that the goat can graze.
Give your answer as a single fraction in its simplest form.
You must show all your working clearly.

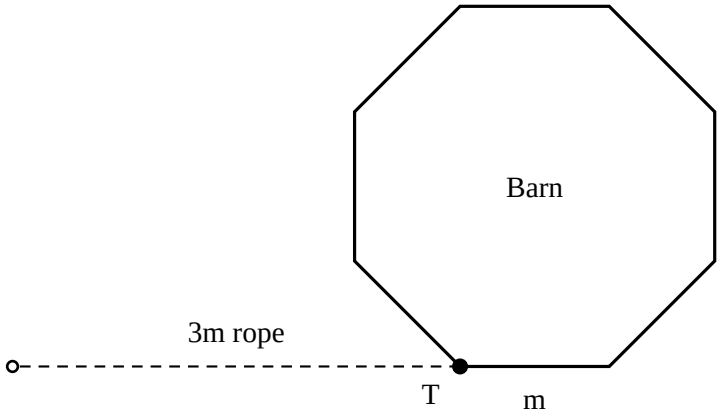


Diagram not to scale. T is the tether point.

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(Total for Question is 6 marks)

I Do - Worked solution and mark scheme

Grey: reachable grass. White polygon: barn. Solid dot: tether.

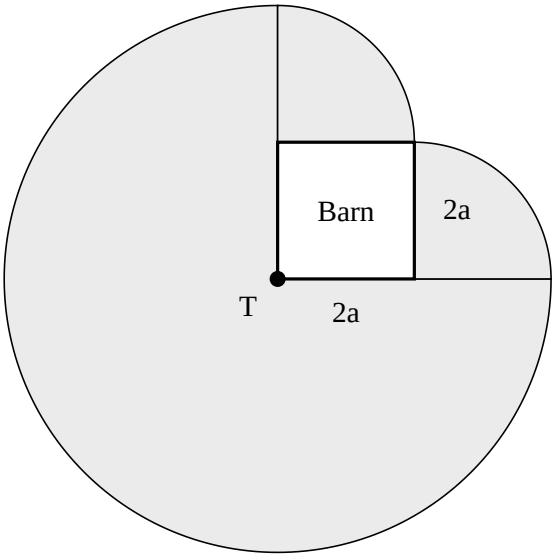


Diagram not to scale. T is the tether point.

Rope wraps follow the walls; each new sector uses only the remaining length.

M1	Identify the 270° main sector and the two possible corner wraps.
M1	Main area $= (270/360) \times \pi \times (4a)^2 = 12\pi a^2$.
M1	Each wrap has radius $4a - 2a = 2a$ and angle 90°.
M1	Two wraps: $2 \times (90/360) \times \pi \times (2a)^2 = 2\pi a^2$. No overlap or further positive-radius wrap.
A1	Total grazing area $= 14\pi a^2$.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

We Do - Worked solution and mark scheme

Grey: reachable grass. White polygon: barn. Solid dot: tether.

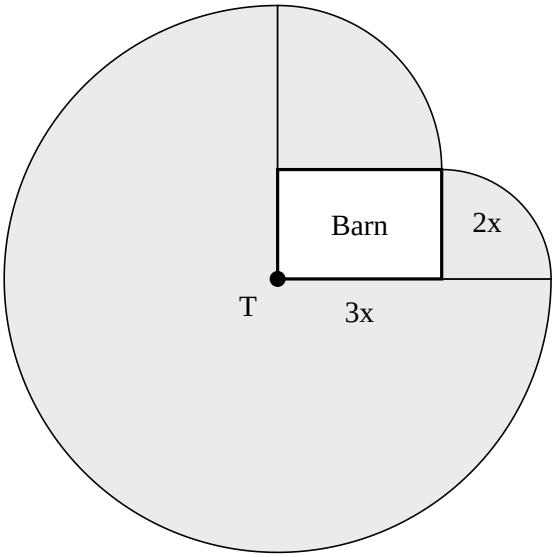


Diagram not to scale. T is the tether point.

Rope wraps follow the walls; each new sector uses only the remaining length.

M1	Main sector angle $= 270^\circ$; area $= (3/4)\pi(5x)^2 = (75/4)\pi x^2$.
M1	After the 3x wall, remaining rope $= 2x$; after the 2x wall, remaining rope $= 3x$.
M1	The two wraps are 90° sectors: $\pi(2x)^2/4$ and $\pi(3x)^2/4$.
M1	Add $(75/4 + 4/4 + 9/4)\pi x^2$; no remaining rope after the next corner.
A1	Total grazing area $= 22\pi x^2$.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

You Do 1 - Worked solution and mark scheme

Grey: reachable grass. White polygon: barn. Solid dot: tether.

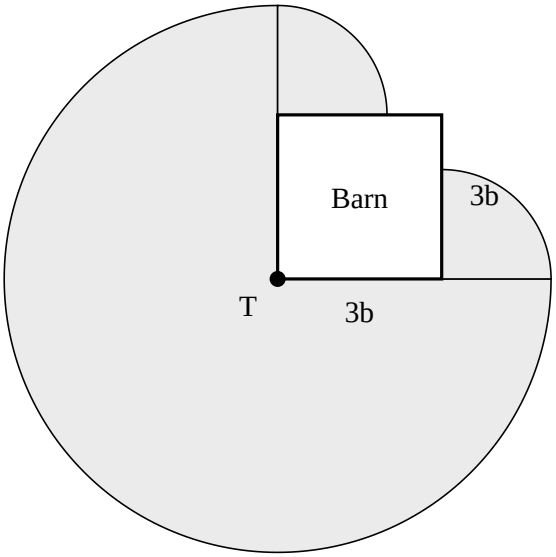


Diagram not to scale. T is the tether point.

Rope wraps follow the walls; each new sector uses only the remaining length.

M1	Main sector: $(3/4)\pi(5b)^2 = (75/4)\pi b^2$.
M1	Both remaining lengths are $5b - 3b = 2b$, each giving a 90° sector.
M1	Two wraps contribute $2 \times (1/4)\pi(2b)^2 = 2\pi b^2$. No further wrap.
A1	Total grazing area $= (83/4)\pi b^2$.

Total: 4 marks. M = method; A = accuracy. Equivalent valid methods accepted.

You Do 2 - Worked solution and mark scheme

Grey: reachable grass. White polygon: barn. Solid dot: tether.

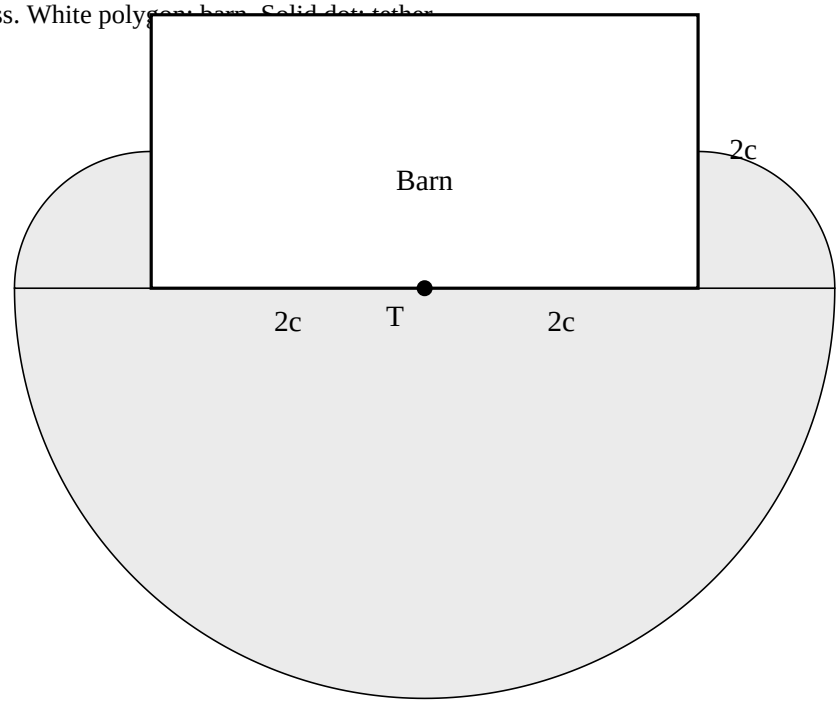


Diagram not to scale. T is the tether point.

Rope wraps follow the walls; each new sector uses only the remaining length.

M1	A tether on a straight wall gives a 180° main sector.
M1	Main area $= (1/2)\pi(3c)^2 = (9/2)\pi c^2$.
M1	The tether is 2c from either end; each wrap radius $= 3c - 2c = c$.
M1	Two 90° sectors contribute $2 \times (1/4)\pi c^2 = (1/2)\pi c^2$. They cannot reach the far corners.
A1	Total grazing area $= 5\pi c^2$.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

You Do 3 - Worked solution and mark scheme

Grey: reachable grass. White polygon: barn. Solid dot: tether.

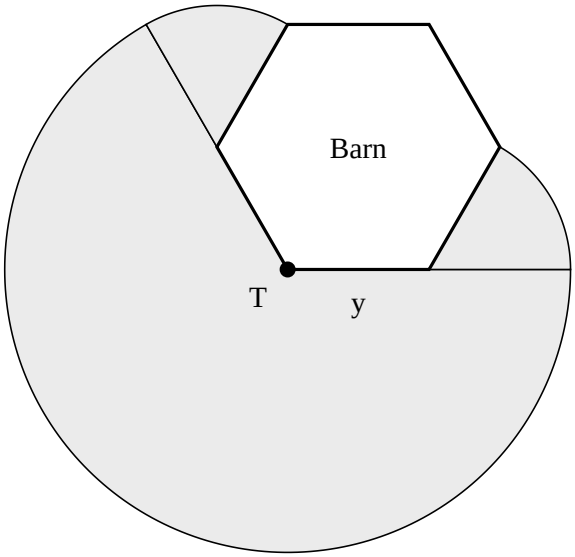


Diagram not to scale. T is the tether point.

Rope wraps follow the walls; each new sector uses only the remaining length.

M1	Hexagon interior angle $= 120^\circ$; initial grazing angle $= 360^\circ - 120^\circ = 240^\circ$.
M1	Main area $= (240/360)\pi(2y)^2 = (8/3)\pi y^2$.
M1	Each adjacent corner leaves radius y; the wrap angle is the exterior angle 60°, not 120°.
M1	Two wraps contribute $2 \times (60/360)\pi y^2 = (1/3)\pi y^2$. No rope remains at the next corners.
A1	Total grazing area $= 3\pi y^2$.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

Challenge 1 - Worked solution and mark scheme

Grey: reachable grass. White polygon: barn. Solid dot: tether.

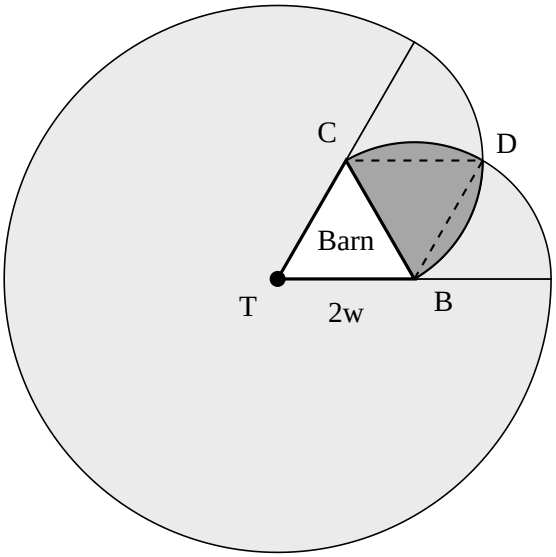


Diagram not to scale. T is the tether point.

Darker grey is the overlap: include it once, not twice.

M1	Triangle interior angle = 60°. Main area = $(300/360)\pi(4w)^2 = (40/3)\pi w^2$.
M1	Two 120° wraps of radius 2w contribute $(8/3)\pi w^2$, but their shaded regions overlap beyond wall BC.
M1	BCD is equilateral, side 2w. Overlap = two 60° sectors minus triangle BCD = $[2 \times (60/360)\pi(2w)^2] - \sqrt{3}w^2 = (4\pi/3 - \sqrt{3})w^2$.
M1	Subtract the darker overlap ONCE: $16\pi w^2 - (4\pi/3 - \sqrt{3})w^2$.
A1	Correct total = $(44\pi/3 + \sqrt{3})w^2$. The uploaded answer $16\pi w^2$ counted the overlap twice.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

Challenge 2 - Worked solution and mark scheme

Grey: reachable grass. White polygon: barn. Solid dot: tether.

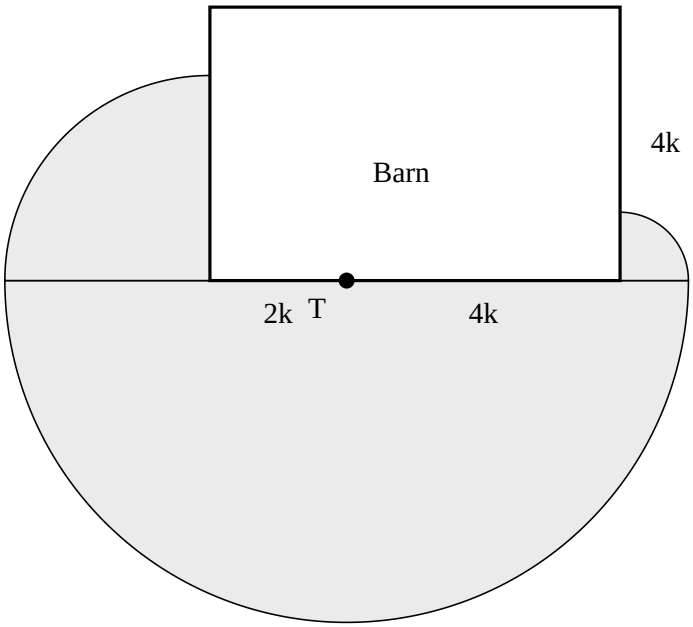


Diagram not to scale. T is the tether point.

Rope wraps follow the walls; each new sector uses only the remaining length.

M1	A wall tether gives a semicircle: area $= (1/2)\pi(5k)^2 = (25/2)\pi k^2$.
M1	Distances to the two corners are 2k and $6k - 2k = 4k$.
M1	Remaining radii are 3k and k. Both are shorter than the 4k side, so there are no further wraps.
M1	Quarter-circle areas sum to $(1/4)\pi[(3k)^2 + k^2] = (5/2)\pi k^2$.
A1	Total grazing area $= 15\pi k^2$.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

Extreme Challenge - Worked solution and mark scheme

Grey: reachable grass. White polygon: barn. Solid dot: tether.

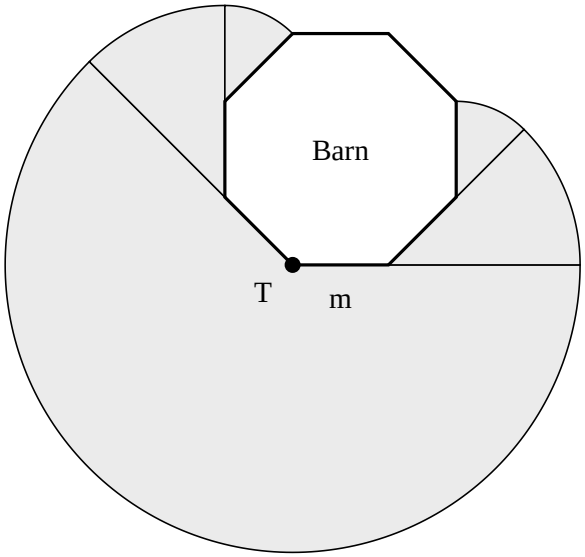


Diagram not to scale. T is the tether point.

Rope wraps follow the walls; each new sector uses only the remaining length.

M1	Octagon interior angle $= 135^\circ$; main angle $= 225^\circ$.
M1	Main area $= (225/360)\pi(3\text{m})^2 = (45/8)\pi\text{m}^2$.
M1	After one wall, each direction leaves 2m; each wrap sweeps the exterior angle 45°.
M1	The first pair contributes $2 \times (45/360)\pi(2\text{m})^2 = \pi\text{m}^2$.
M1	The next pair has radius m and contributes $2 \times (45/360)\pi\text{m}^2 = (1/4)\pi\text{m}^2$. No rope remains after a third wall.
A1	No sector overlap. Total grazing area $= (55/8)\pi\text{m}^2$.

Total: 6 marks. M = method; A = accuracy. Equivalent valid methods accepted.