Loci and Regions: Area of Sectors

Practice Worksheet (Problem Solving)

I Do

A robotic lawn mower is connected to a charging post by a cable of length $4a$.

The charging post is fixed at an outside corner of a square shed with side length $2a$.

The mower cannot pass through the shed.

Find an expression, in terms of a and π, for the maximum area of grass the mower can cut. Give your answer in its simplest form.

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(Total for Question is 4 marks)

We Do

A robotic lawn mower is connected to a charging post by a cable of length $3a$.

The charging post is fixed at an outside corner of a rectangular shed.
The shed has a length of $2a$ and a width of a.
The post is located at a corner connecting the length and the width.

The mower cannot pass through the shed.

Find an expression, in terms of a and π, for the maximum area of grass the mower can cut.
Give your answer in its simplest form.

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(Total for Question is 4 marks)

You Do 1

A robotic lawn mower is connected to a charging post by a cable of length $5a$.

The charging post is fixed at an outside corner of a square shed with side length $3a$.

The mower cannot pass through the shed.

Find an expression, in terms of a and π, for the maximum area of grass the mower can cut.
Give your answer as a single fraction in its simplest form.

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(Total for Question is 4 marks)

You Do 2

A robotic lawn mower is connected to a charging post by a cable of length $2a$.

The charging post is fixed at an outside corner of a square shed with side length $2a$.

The mower cannot pass through the shed.

Find an expression, in terms of a and π, for the maximum area of grass the mower can cut. Give your answer in its simplest form.

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(Total for Question is 3 marks)

You Do 3

A robotic lawn mower is connected to a charging post by a cable of length $2a$.

The charging post is fixed at an outside corner of a shed shaped like an equilateral triangle with side length $2a$.

The mower cannot pass through the shed.

Find an expression, in terms of a and π, for the maximum area of grass the mower can cut. Give your answer as a single fraction in its simplest form.

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(Total for Question is 4 marks)

Challenge 1

A robotic lawn mower is connected to a charging post by a cable of length $3a$.

The charging post is fixed at an outside corner of a shed shaped like an equilateral triangle with side length $2a$.

The mower cannot pass through the shed.

Find an expression, in terms of a and π, for the maximum area of grass the mower can cut.
Give your answer as a single fraction in its simplest form.
Show clear geometric reasoning.

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(Total for Question is 5 marks)

Challenge 2

A shed is built in the shape of a triangle with side lengths $3a$, $5a$, and $7a$.

A robotic lawn mower is connected to a charging post fixed at the outside corner where the $3a$ wall meets the $5a$ wall.

The mower is attached by a cable of length $3a$.
The mower cannot pass through the shed.

Use trigonometry to find an expression, in terms of a and π, for the maximum area of grass the mower can cut.
Give your answer in its simplest form.

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(Total for Question is 5 marks)

Extreme Challenge

A shed is built in the shape of a regular hexagon with side length $2a$.

A robotic lawn mower is connected to a charging post fixed at one of the outside corners of the shed.

The mower is attached by a cable of length $5a$.
The mower cannot pass through the shed.

Find an expression, in terms of a and π, for the maximum area of grass the mower can cut.
Give your answer in its simplest form.
You must show all your working clearly.

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(Total for Question is 6 marks)

Worked Answers

I Do

The interior angle of the square shed is $90°$.
The reflex exterior angle where the post is attached is $360° - 90° = 270°$.
Main sweeping sector: A $\frac{3}{4}$ circle with radius $4a$.
Area 1 $= \frac{3}{4} \times \pi \times (4a)^2 = \frac{3}{4} \times 16\pi a^2 = 12\pi a^2$.
The cable wraps around the two adjacent corners of the shed.
Remaining cable length $= 4a - 2a = 2a$.
The exterior angle at a straight edge is $180° - 90° = 90°$.
Two wrap-around sectors: Two $\frac{1}{4}$ circles with radius $2a$.
Area 2 $= 2 \times \frac{1}{4} \times \pi \times (2a)^2 = \frac{1}{2} \times 4\pi a^2 = 2\pi a^2$.
Total Area $= 12\pi a^2 + 2\pi a^2 = \mathbf{14\pi a^2}$.

We Do

The reflex angle at the corner of the rectangle is $270°$.
Main sweeping sector: A $\frac{3}{4}$ circle with radius $3a$.
Area 1 $= \frac{3}{4} \times \pi \times (3a)^2 = \frac{27}{4}\pi a^2$.
Wrap 1 (along the $2a$ side): Remaining cable $= 3a - 2a = a$.
This forms a $\frac{1}{4}$ circle with radius a.
Area 2 $= \frac{1}{4} \times \pi \times (a)^2 = \frac{1}{4}\pi a^2$.
Wrap 2 (along the a side): Remaining cable $= 3a - a = 2a$.
This forms a $\frac{1}{4}$ circle with radius $2a$.
Area 3 $= \frac{1}{4} \times \pi \times (2a)^2 = \pi a^2 = \frac{4}{4}\pi a^2$.
Total Area $= \frac{27}{4}\pi a^2 + \frac{1}{4}\pi a^2 + \frac{4}{4}\pi a^2 = \frac{32}{4}\pi a^2 = \mathbf{8\pi a^2}$.

You Do 1

Main sector: Reflex angle $270°$, radius $5a$.
Area 1 $= \frac{3}{4} \times \pi \times (5a)^2 = \frac{75}{4}\pi a^2$.
Wraps: Cable hits two adjacent corners. Remaining length $= 5a - 3a = 2a$.
Two $\frac{1}{4}$ circles of radius $2a$.
Area 2 $= 2 \times \frac{1}{4} \times \pi \times (2a)^2 = 2\pi a^2 = \frac{8}{4}\pi a^2$.
Total Area $= \frac{75}{4}\pi a^2 + \frac{8}{4}\pi a^2 = \mathbf{\frac{83}{4}\pi a^2}$.

You Do 2

The cable length $(2a)$ is exactly equal to the side length of the shed $(2a)$.
It will not wrap around the corners.
Main sector only: Reflex angle $270°$, radius $2a$.
Area $= \frac{3}{4} \times \pi \times (2a)^2 = \frac{3}{4} \times 4\pi a^2 = \mathbf{3\pi a^2}$.

You Do 3

The interior angle of an equilateral triangle is $60°$.
Reflex angle at the post $= 360° - 60° = 300°$.
The cable length $(2a)$ equals the side length, so there are no wrap-around sectors.
Area $= \frac{300}{360} \times \pi \times (2a)^2 = \frac{5}{6} \times 4\pi a^2 = \mathbf{\frac{10}{3}\pi a^2}$.

Challenge 1

Reflex angle at the post $= 300°$. Main sector radius $= 3a$.
Area 1 $= \frac{300}{360} \times \pi \times (3a)^2 = \frac{5}{6} \times 9\pi a^2 = \frac{15}{2}\pi a^2$.
The cable $(3a)$ hits two adjacent corners of the shed $(2a)$.
Remaining cable $= 3a - 2a = a$.
The angle it sweeps around the corner is the exterior angle: $180° - 60° = 120°$.
There are two of these sectors.
Area 2 $= 2 \times \frac{120}{360} \times \pi \times (a)^2 = 2 \times \frac{1}{3}\pi a^2 = \frac{2}{3}\pi a^2$.
Total Area $= \frac{15}{2}\pi a^2 + \frac{2}{3}\pi a^2 = \frac{45}{6}\pi a^2 + \frac{4}{6}\pi a^2 = \mathbf{\frac{49}{6}\pi a^2}$.

Challenge 2

First, use the Cosine Rule to find the interior angle θ between the $3a$ and $5a$ sides:
$\cos\theta = \frac{(3a)^2+(5a)^2-(7a)^2}{2(3a)(5a)} = \frac{9a^2+25a^2-49a^2}{30a^2} = \frac{-15a^2}{30a^2} = -0.5$.
Since $\cos\theta = -0.5$, the interior angle $\theta = 120°$.
The reflex angle at the charging post $= 360° - 120° = 240°$.
The cable has length $3a$. Because the shortest adjacent wall is $3a$, the cable will exactly reach the corner but will not wrap around it (remaining length is 0). It does not reach the end of the $5a$ wall.
So, the maximum area is just the main sweeping sector.
Area $= \frac{240}{360} \times \pi \times (3a)^2 = \frac{2}{3} \times 9\pi a^2 = \mathbf{6\pi a^2}$.

Extreme Challenge

The interior angle of a regular hexagon is $\frac{(6-2)\times 180}{6} = 120°$.
Reflex angle at the post $= 360° - 120° = 240°$.
Main sector radius $= 5a$.
Area 1 $= \frac{240}{360} \times \pi \times (5a)^2 = \frac{2}{3} \times 25\pi a^2 = \frac{50}{3}\pi a^2$.
Wrap 1: The cable $(5a)$ wraps around two adjacent corners (side $2a$).
Remaining cable $= 5a - 2a = 3a$.
Sweep angle is the exterior angle: $180° - 120° = 60°$.
Area 2 $= 2 \times \frac{60}{360} \times \pi \times (3a)^2 = 2 \times \frac{1}{6} \times 9\pi a^2 = 3\pi a^2$.
Wrap 2: The remaining cable $(3a)$ wraps around the next two corners (side $2a$).
Remaining cable $= 3a - 2a = a$.
Sweep angle remains the exterior angle: $60°$.
Area 3 $= 2 \times \frac{60}{360} \times \pi \times (a)^2 = 2 \times \frac{1}{6}\pi a^2 = \frac{1}{3}\pi a^2$.
Total Area $= \frac{50}{3}\pi a^2 + 3\pi a^2 + \frac{1}{3}\pi a^2 = \frac{50}{3}\pi a^2 + \frac{9}{3}\pi a^2 + \frac{1}{3}\pi a^2 = \frac{60}{3}\pi a^2 = \mathbf{20\pi a^2}$.