Overlapping Squares and Rectangles

Practice Worksheet (Problem Solving)

I Do

ABCD is a rectangle. PQRS is a square with P on AB, R on BC and S on CD. Q lies outside the rectangle. T is where QR crosses AB. AB = 8 cm, BC = 4 cm and PQ = 5 cm. The shaded region is inside both shapes. Calculate its exact area. Show all your working.

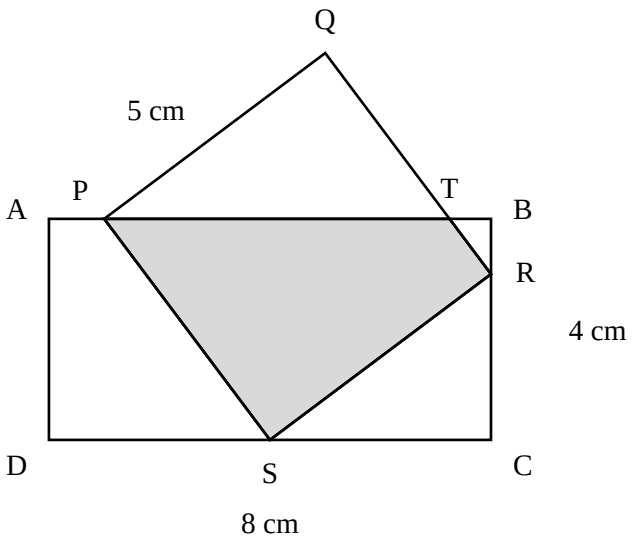


Diagram not to scale. T is where QR crosses AB.

Video walkthrough

(Total for Question is 5 marks)

We Do

ABCD is a rectangle. PQRS is a square with P on AB, R on BC and S on CD. Q lies outside the rectangle. T is where QR crosses AB. AB = 16 cm, BC = 8 cm and PQ = 10 cm. The shaded region is inside the rectangle but outside the square. Calculate its area.

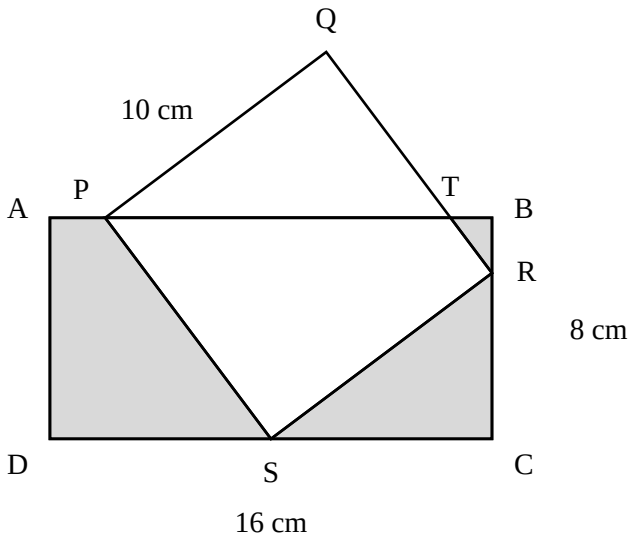


Diagram not to scale. T is where QR crosses AB.

(Total for Question is 5 marks)

You Do 1

ABCD is a rectangle. PQRS is a square with P on AB, R on BC and S on CD. Q lies outside the rectangle. T is where QR crosses AB. AB = 24 cm, BC = 12 cm and PQ = 15 cm. Calculate the shaded overlap area. Give your answer as an exact fraction.

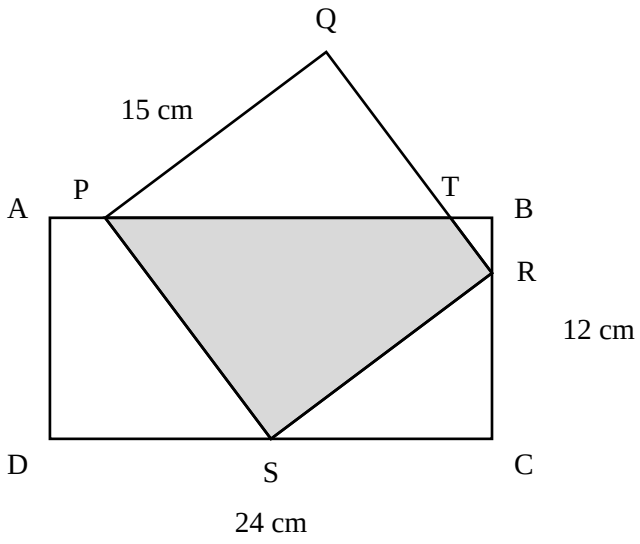


Diagram not to scale. T is where QR crosses AB.

(Total for Question is 4 marks)

You Do 2

ABCD is a rectangle. PQRS is a square with P on AB, R on BC and S on CD. Q lies outside the rectangle. T is where QR crosses AB. AB = 12 cm, BC = 6 cm and PQ = $3\sqrt{5}$ cm. The shaded region is inside the square but outside the rectangle. Calculate its exact area.

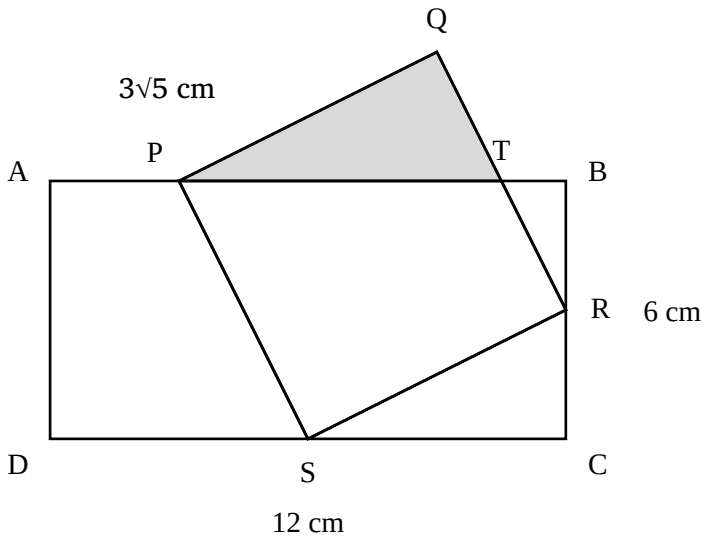


Diagram not to scale. T is where QR crosses AB.

(Total for Question is 5 marks)

You Do 3

ABCD is a rectangle. PQRS is a square with P on AB, R on BC and S on CD. Q lies outside the rectangle. T is where QR crosses AB. AB = 10 cm, BC = 5 cm and PQ = √34 cm. Calculate the shaded area inside the rectangle but outside the square.

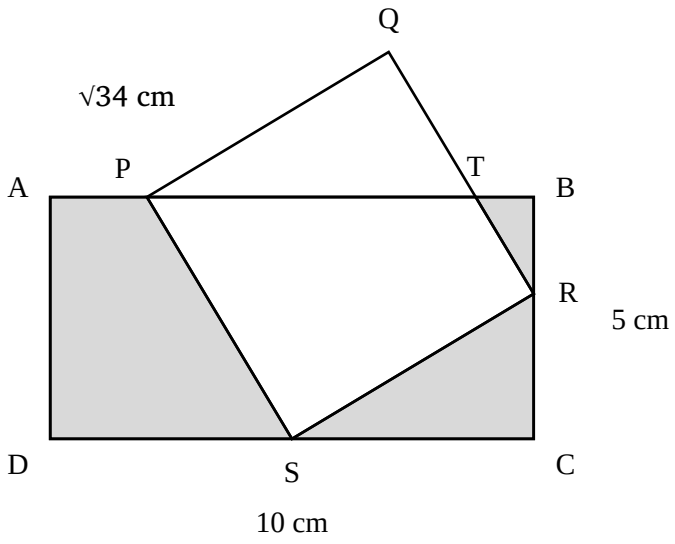


Diagram not to scale. T is where QR crosses AB.

(Total for Question is 5 marks)

Challenge 1

ABCD is a rectangle. PQRS is a square with P on AB, R on BC and S on CD. Q lies outside the rectangle. T is where QR crosses AB. AB = 11 cm, BC = 5 cm and PQ = 6 cm. Calculate the shaded overlap area. Give your answer in the form a − b√c, where a, b and c are rational numbers.

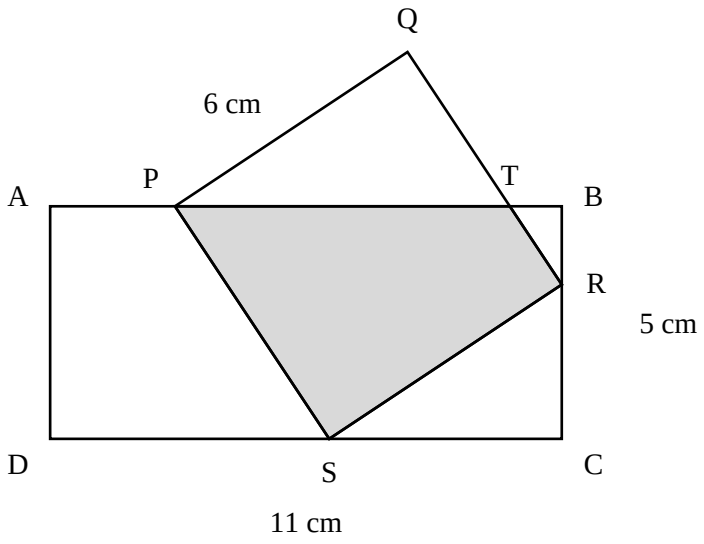


Diagram not to scale. T is where QR crosses AB.

(Total for Question is 5 marks)

Challenge 2

ABCD is a rectangle. PQRS is a square with P on AB, R on BC and S on CD. Q lies outside the rectangle. T is where QR crosses AB. This arrangement is a similar enlargement of the I Do diagram. Its shaded overlap has area 40 cm^2. Calculate the area of the square. You do not need to find any lengths.

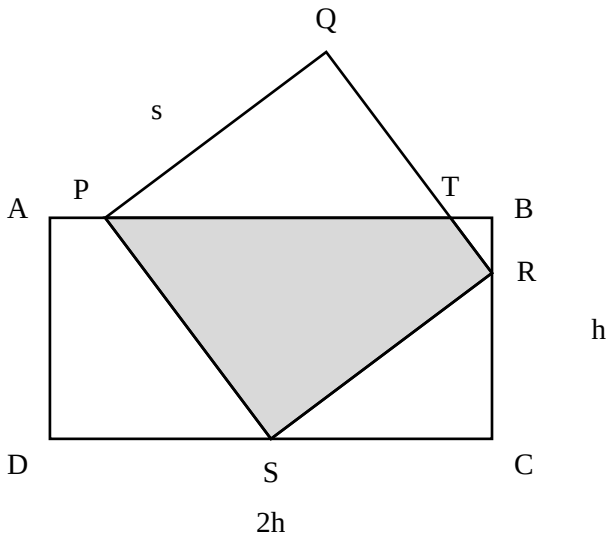


Diagram not to scale. T is where QR crosses AB.

(Total for Question is 5 marks)

Extreme Challenge

ABCD is a rectangle. PQRS is a square with P on AB, R on BC and S on CD. Q lies outside the rectangle. T is where QR crosses AB. Let BC = h and AB = W, where W > 3h/2. The horizontal projection d of PS is h/2. Show that the shaded overlap area is $15h^2/16$ and is three quarters of the square's area. Explain why W does not appear in the result.

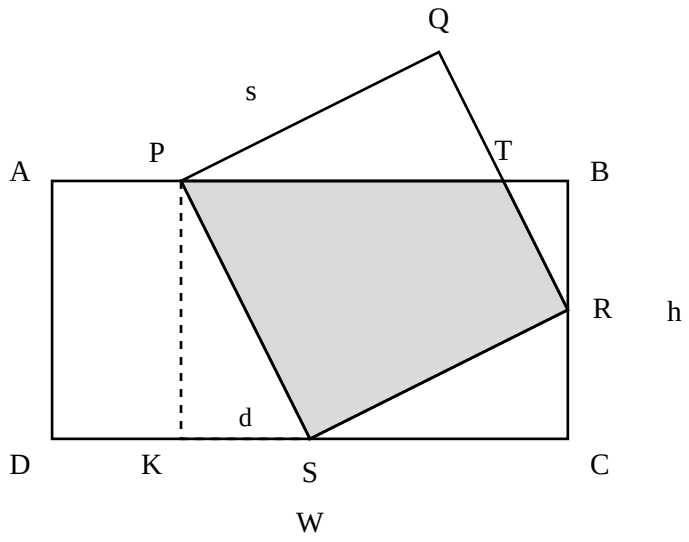


Diagram not to scale. T is where QR crosses AB.

(Total for Question is 6 marks)

I Do - Worked solution and mark scheme

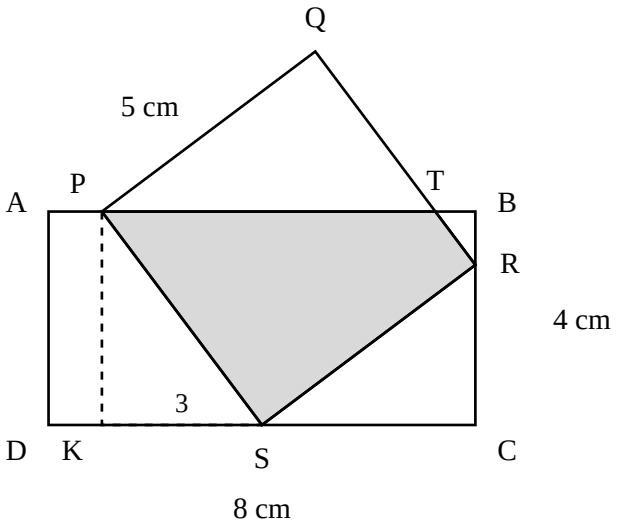


Diagram not to scale. T is where QR crosses AB.

M1	Let the horizontal projection of PS be d. Pythagoras gives $d^2 + 4^2 = 5^2$, so $d = 3$ cm.
M1	The right triangles determined by the square have perpendicular side directions. The rise from P to Q is 3 cm; Q is 3 cm above AB.
M1	$PB = 4 + 3 = 7$ cm and $RB = 4 - 3 = 1$ cm. Similar triangles give $BT/RB = 3/4$, so $BT = 3/4$ cm and $PT = 7 - 3/4 = 25/4$ cm.
M1	Area outside the rectangle is triangle PQT: $(1/2) \times (25/4) \times 3 = 75/8$ cm^2.
A1	Shaded overlap $= 25 - 75/8 = 125/8$ cm^2 (15.625 cm^2).

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

We Do - Worked solution and mark scheme

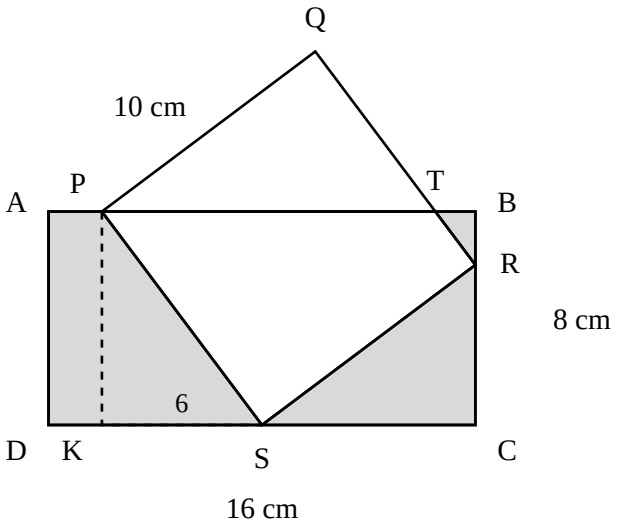


Diagram not to scale. T is where QR crosses AB.

M1	Pythagoras gives d = $\sqrt{(10^2 - 8^2)}$ = 6 cm.
M1	The top triangle has height 6 cm and base PT = $10^2/8$ = 25/2 cm.
M1	Top triangle area = (1/2) × (25/2) × 6 = 75/2 cm^2.
M1	Overlap = 100 − 75/2 = 125/2 cm^2; rectangle area = 16 × 8 = 128 cm^2.
A1	Shaded area = 128 − 125/2 = 131/2 cm^2 (65.5 cm^2).

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

You Do 1 - Worked solution and mark scheme

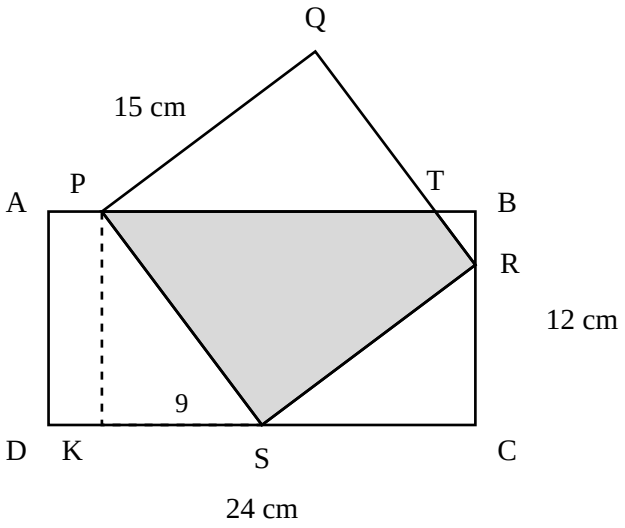


Diagram not to scale. T is where QR crosses AB.

M1	Pythagoras: $d = \sqrt{(15^2 - 12^2)} = 9$ cm.
M1	$PT = 15^2/12 = 75/4$ cm. Top triangle area $= (1/2) \times (75/4) \times 9 = 675/8$ cm^2.
M1	Subtract the top triangle from square area $15^2 = 225$ cm^2.
A1	Overlap $= 225 - 675/8 = 1125/8$ cm^2.

Total: 4 marks. M = method; A = accuracy. Equivalent valid methods accepted.

You Do 2 - Worked solution and mark scheme

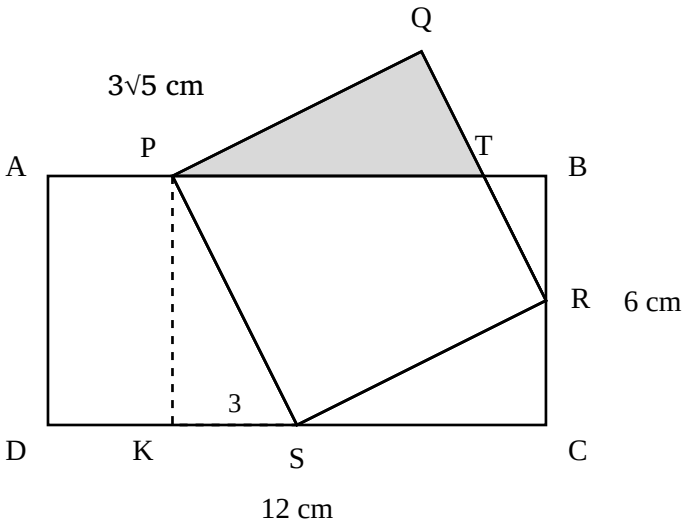


Diagram not to scale. T is where QR crosses AB.

M1	Square area $= (3\sqrt{5})^2 = 45$ cm^2.
M1	Pythagoras gives $d^2 = 45 - 36 = 9$, so $d = 3$ cm.
M1	By similar triangles $PT = 45/6 = 15/2$ cm.
M1	The shaded region is the top triangle PQT, height 3 cm.
A1	Area $= (1/2) \times (15/2) \times 3 = 45/4$ cm^2.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

You Do 3 - Worked solution and mark scheme

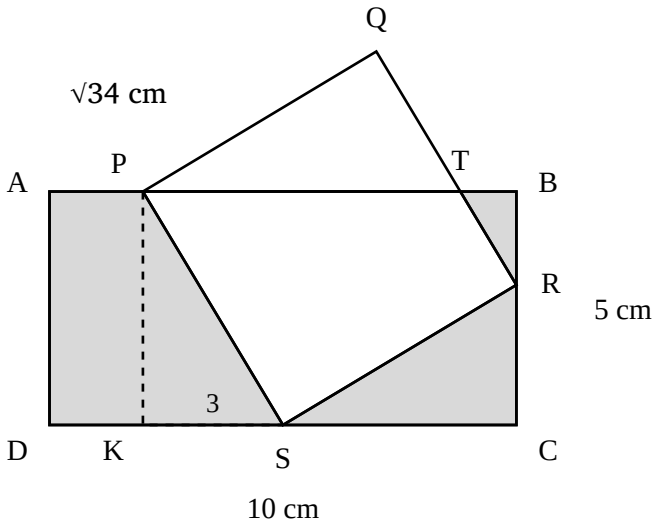


Diagram not to scale. T is where QR crosses AB.

M1	Pythagoras gives $d = \sqrt{(34 - 25)} = 3$ cm.
M1	$PT = 34/5$ cm; the top triangle height is 3 cm.
M1	Top triangle area $= (1/2) \times (34/5) \times 3 = 51/5$ cm^2.
M1	Overlap $= 34 - 51/5 = 119/5$ cm^2.
A1	Rectangle area is 50 cm^2. Shaded area $= 50 - 119/5 = 131/5$ cm^2.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

Challenge 1 - Worked solution and mark scheme

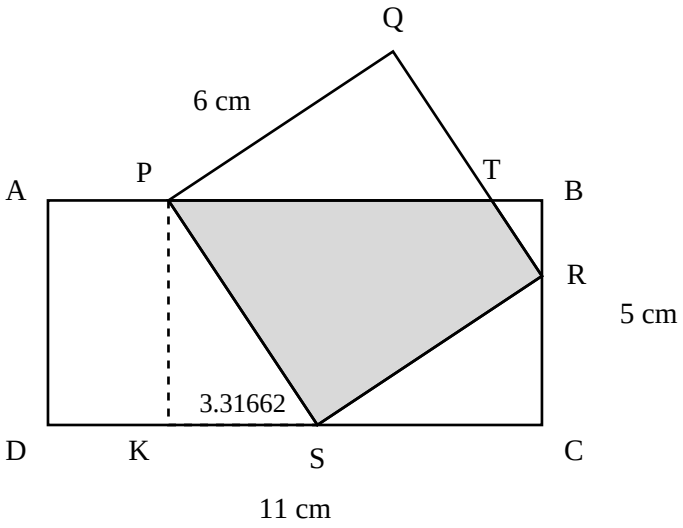


Diagram not to scale. T is where QR crosses AB.

M1	Pythagoras: $d^2 = 6^2 - 5^2 = 11$, so $d = \sqrt{11}$ cm.
M1	The top triangle has height $\sqrt{11}$ cm.
M1	Similar triangles give $PT = 6^2/5 = 36/5$ cm.
M1	Top triangle area $= (1/2) \times (36/5) \times \sqrt{11} = (18/5)\sqrt{11}$ cm^2.
A1	Overlap $= 36 - (18/5)\sqrt{11}$ cm^2.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

Challenge 2 - Worked solution and mark scheme

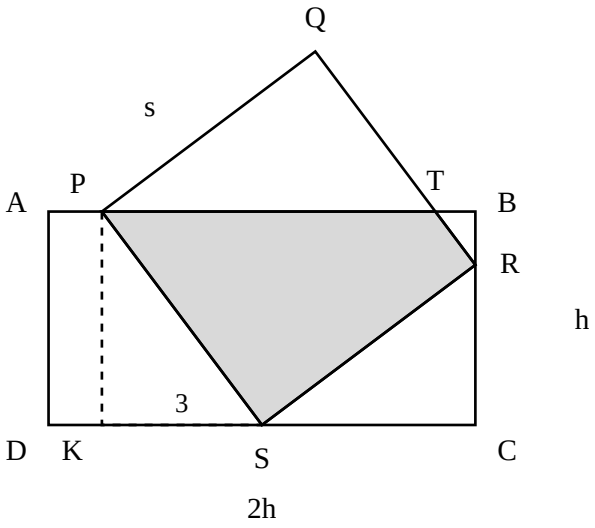


Diagram not to scale. T is where QR crosses AB.

M1	In the I Do diagram, square area is 25 cm^2 and overlap area is 125/8 cm^2.
M1	The overlap-to-square area ratio is (125/8)/25 = 5/8.
M1	Similar enlargements preserve this ratio of areas; it is not a ratio of lengths.
M1	If square area is A, then (5/8)A = 40.
A1	A = 40 × 8/5 = 64 cm^2.

Total: 5 marks. M = method; A = accuracy. Equivalent valid methods accepted.

Extreme Challenge - Worked solution and mark scheme

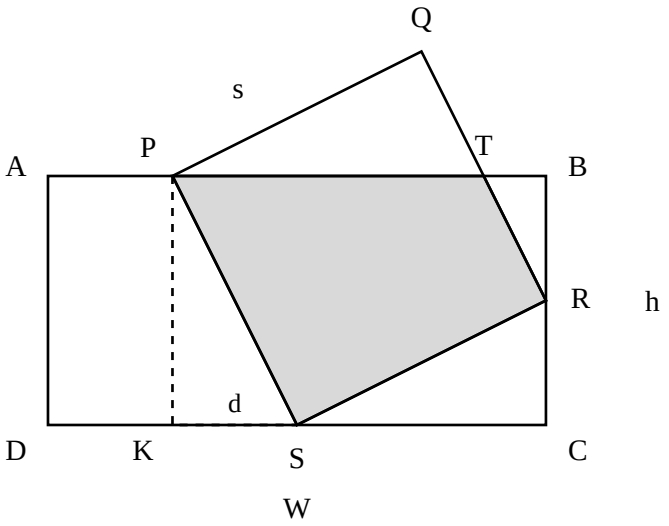


Diagram not to scale. T is where QR crosses AB.

M1	Pythagoras gives $s^2 = h^2 + d^2 = h^2 + (h/2)^2 = 5h^2/4$.
M1	The rise from P to Q is $d = h/2$, so the top triangle has height $h/2$.
M1	$PB = h + d$ and $BT = d(h - d)/h$, so $PT = PB - BT = h + d^2/h = s^2/h = 5h/4$.
M1	Top triangle area $= (1/2) \times (5h/4) \times (h/2) = 5h^2/16$.
M1	Overlap $= 5h^2/4 - 5h^2/16 = 15h^2/16$. Divide by $5h^2/4$ to obtain 3/4.
A1	W only sets the horizontal position of the square. Once the contacts and h are fixed, the square and the removed triangle are unchanged.

Total: 6 marks. M = method; A = accuracy. Equivalent valid methods accepted.