Angles in Polygons

Practice Worksheet (Problem Solving)

I Do

Each exterior angle of a regular polygon is $15°$.

(a) Work out how many sides the polygon has.

..... *(2 marks)*

(b) Work out the size of each interior angle.

.....° *(1 mark)*

(Total for Question is 3 marks)

We Do

Each exterior angle of a regular polygon is $24°$.

(a) Work out how many sides the polygon has.

..... *(2 marks)*

(b) Work out the size of each interior angle.

.....° *(1 mark)*

(Total for Question is 3 marks)

You Do 1

Each interior angle of a regular polygon is $160°$.

(a) Work out the size of each exterior angle.

.....° *(1 mark)*

(b) Work out how many sides the polygon has.

..... *(2 marks)*

(Total for Question is 3 marks)

You Do 2

A regular polygon has 12 sides.

(a) Work out the size of each exterior angle.

.....° *(1 mark)*

(b) Work out the sum of the interior angles of the polygon.

.....° *(2 marks)*

(Total for Question is 3 marks)

You Do 3

The sum of the interior angles of a regular polygon is $1440°$.

(a) Work out how many sides the polygon has.

..... *(2 marks)*

(b) Work out the size of each exterior angle.

.....° *(1 mark)*

(Total for Question is 3 marks)

Challenge 1

For a regular polygon, the ratio of the size of an interior angle to the size of an exterior angle is $7 : 2$.

Work out how many sides the polygon has.
Show all of your working clearly.

.....

(Total for Question is 4 marks)

Challenge 2

The diagram shows a square, a regular hexagon, and part of a regular n-sided polygon meeting at a point B.

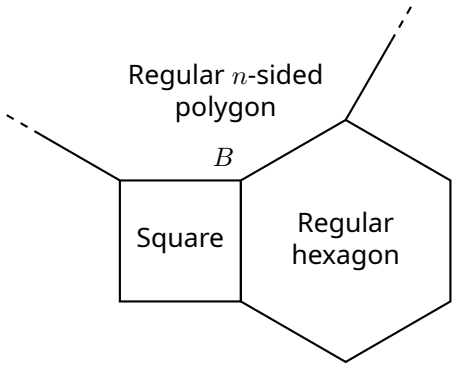


Diagram **NOT** accurately drawn

Work out the value of n.
Show all of your working clearly.

$n =$ .....

(Total for Question is 4 marks)

Extreme Challenge

Polygon A is a regular polygon with n sides.
Polygon B is a regular polygon with $2n$ sides.

Each interior angle of Polygon B is $12°$ greater than each interior angle of Polygon A.

(a) Work out the value of n.
You must show all of your algebraic working clearly.

$n =$ ..... *(3 marks)*

(b) Work out the sum of the interior angles of Polygon B.

.....° *(2 marks)*

(Total for Question is 5 marks)

Worked Answers

I Do
(a) The sum of the exterior angles of any polygon is $360°$.
Number of sides $= \frac{360°}{15°} =$ **24**.
(b) Interior angle + Exterior angle $= 180°$.
Each interior angle $= 180° - 15° =$ **165**°.

We Do
(a) Number of sides $= \frac{360°}{24°} =$ **15**.
(b) Each interior angle $= 180° - 24° =$ **156**°.

You Do 1
(a) Each exterior angle $= 180° - 160° =$ **20**°.
(b) Number of sides $= \frac{360°}{20°} =$ **18**.

You Do 2
(a) Each exterior angle $= \frac{360°}{12} =$ **30**°.
(b) Sum of interior angles $= (n-2) \times 180° = (12-2) \times 180° = 10 \times 180° =$ **1800**°.
(Alternatively, each interior angle is $180° - 30° = 150°$, so sum $= 12 \times 150° = 1800°$.)

You Do 3
(a) Use the sum of interior angles formula:
$(n-2) \times 180 = 1440$
$n - 2 = \frac{1440}{180} = 8$
n $=$ **10**.
(b) Each exterior angle $= \frac{360°}{10} =$ **36**°.

Challenge 1
1. An interior angle and its adjacent exterior angle lie on a straight line, so they sum to $180°$.
2. Total ratio parts $= 7 + 2 = 9$.
3. Value of 1 part $= \frac{180°}{9} = 20°$.
4. Size of each exterior angle $= 2 \times 20° = 40°$.
5. Number of sides $= \frac{360°}{40°} =$ **9**.

Challenge 2

1. Find the interior angle of the square: $90°$.
2. Find the interior angle of the regular hexagon:
Exterior angle $= \frac{360°}{6} = 60° \implies$ Interior angle $= 180° - 60° = 120°$.
3. Angles around point B sum to $360°$, so the interior angle of the regular n-sided polygon is:
$360° - 90° - 120° = 150°$.
4. Find the exterior angle of the regular n-sided polygon:
$180° - 150° = 30°$.
5. Find the number of sides n:
$n = \frac{360°}{30°} =$ **12**.

Extreme Challenge

(a) 1. Write expressions for the exterior angle of each polygon:
Exterior angle of Polygon $A = \frac{360}{n}$.
Exterior angle of Polygon $B = \frac{360}{2n} = \frac{180}{n}$.
2. Since each interior angle of Polygon B is $12°$ greater than each interior angle of Polygon A, each exterior angle of Polygon B is $12°$ smaller than each exterior angle of Polygon A:
$\frac{360}{n} - \frac{180}{n} = 12$
(Equivalently, using interior angles: $\left(180 - \frac{360}{2n}\right) - \left(180 - \frac{360}{n}\right) = 12$*.)*
3. Simplify and solve for n:
$\frac{180}{n} = 12$
$12n = 180 \implies$ **n** $=$ **15**.

(b) Polygon B has $2n = 2 \times 15 = 30$ sides.
Sum of interior angles $= (30 - 2) \times 180° = 28 \times 180° =$ **5040**°.