Circle Theorems

Practice Worksheet (Problem Solving)

I Do

A, B, and C are points on the circumference of a circle, centre O.
PA and PB are tangents to the circle from point P.
Angle $APB = 56°$.

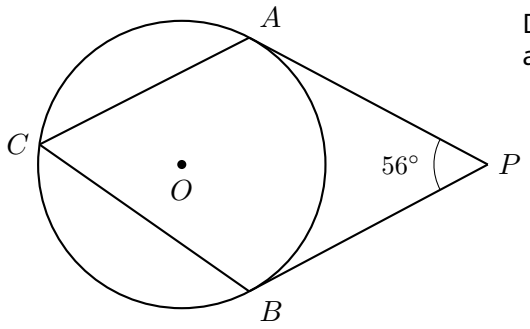


Diagram **NOT** accurately drawn

Work out the size of angle ACB.
You must give a reason for each stage of your working.

.....°

(Total for Question is 4 marks)

We Do

A, B, and C are points on the circumference of a circle, centre O.
AB is a diameter of the circle.
Angle $CAB = 32°$.

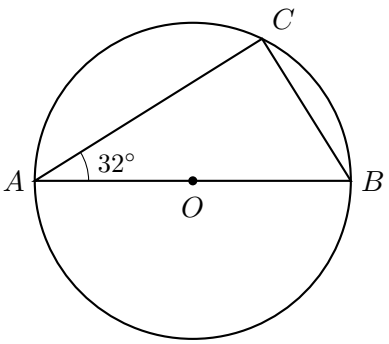


Diagram **NOT** accurately drawn

Work out the size of angle ABC.
You must give a reason for each stage of your working.

.....°

(Total for Question is 3 marks)

You Do 1

A, B, C, and D are points on the circumference of a circle.
The chords AC and BD intersect at the point E.
Angle $DAC = 38°$ and angle $AEB = 104°$.

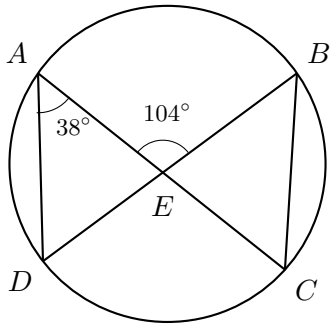


Diagram **NOT** accurately drawn

Work out the size of angle ACB.
You must give a reason for each stage of your working.

.....°

(Total for Question is 3 marks)

You Do 2

A, B, C, and D are points on the circumference of a circle.
ADE is a straight line and $AB = BC$.
Angle $CDE = 68°$.

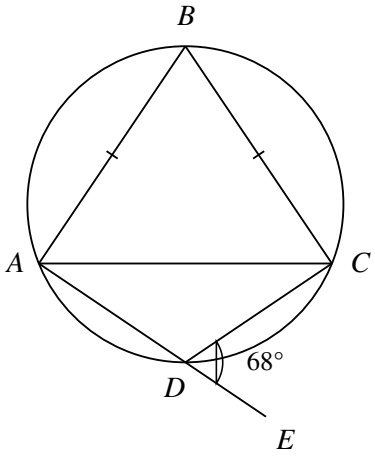


Diagram **NOT** accurately drawn

Work out the size of angle BAC.
You must give a reason for each stage of your working.

.....°

(Total for Question is 4 marks)

You Do 3

A, B, and C are points on the circumference of a circle, centre O.
Angle $ABC = 118°$.

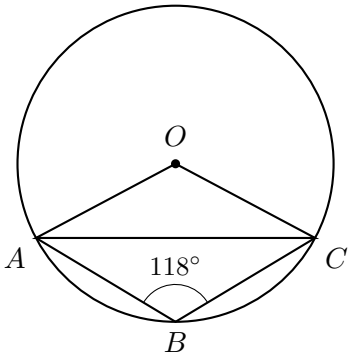


Diagram **NOT** accurately drawn

Work out the size of angle OAC.
You must give a reason for each stage of your working.

.....°

(Total for Question is 4 marks)

Challenge 1

A, B, and C are points on the circumference of a circle, centre O, with radius 10 cm.
AB is a chord of length 16 cm, and M is the midpoint of AB.
The straight line OMC is a radius of the circle.

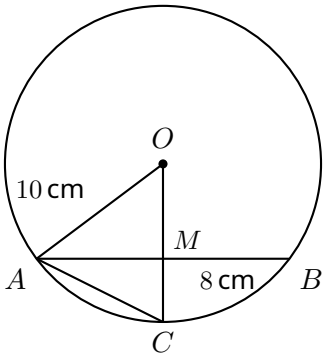


Diagram **NOT** accurately drawn

Work out the exact length of AC.
Give your answer in the form $a\sqrt{b}$, where a and b are integers.
You must give a geometric reason for your first step.

.....cm

(Total for Question is 4 marks)

Challenge 2

A, B, C, and D are points on the circumference of a circle, centre O.
Angle $AOC = (2x + 40)°$ and angle $ABC = (3x - 20)°$.

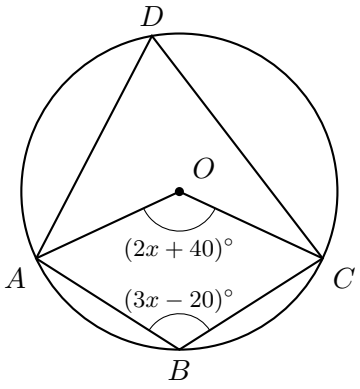


Diagram **NOT** accurately drawn

Work out the value of x, and hence find the size of angle ADC.
You must give a reason for each stage of your working.

$x =$ ..... Angle $ADC =$ .....°

(Total for Question is 5 marks)

Extreme Challenge

A, B, and C are points on the circumference of a circle, centre O.
AB is a diameter of the circle.
PA and PC are tangents to the circle from point P, and angle $APC = 2x$.

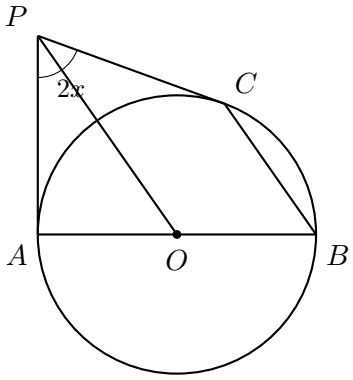


Diagram **NOT** accurately drawn

Prove that the line OP is parallel to the chord BC.
You must give a reason for each stage of your proof.

(Total for Question is 5 marks)

Worked Answers

I Do
1. Draw radii OA and OB.
$\angle OAP = 90°$ and $\angle OBP = 90°$ *(a tangent is perpendicular to the radius at the point of contact)*.
2. Angles in quadrilateral $OAPB$ sum to $360°$:
$\angle AOB = 360° - 90° - 90° - 56° = 124°$.
3. Use the angle at the centre theorem:
$\angle ACB = 124° \div 2 =$ **62**° *(the angle at the centre is twice the angle at the circumference)*.

We Do
1. Since AB is a diameter, $\angle ACB = 90°$ *(the angle in a semicircle is $90°$)*.
2. Angles in triangle ABC sum to $180°$:
$\angle ABC = 180° - 90° - 32° =$ **58**°.

You Do 1
1. Angles on a straight line add up to $180°$:
$\angle AED = 180° - 104° = 76°$.
2. Angles in triangle ADE add up to $180°$:
$\angle ADB = 180° - 76° - 38° = 66°$.
3. $\angle ACB = \angle ADB =$ **66**° *(angles in the same segment are equal)*.

You Do 2
1. Angles on a straight line add up to $180°$:
$\angle ADC = 180° - 68° = 112°$.
2. $ABCD$ is a cyclic quadrilateral:
$\angle ABC = 180° - 112° = 68°$ *(opposite angles in a cyclic quadrilateral add up to $180°$)*.
3. Since $AB = BC$, triangle ABC is isosceles *(base angles of an isosceles triangle are equal)*:
$\angle BAC = (180° - 68°) \div 2 = 112° \div 2 =$ **56**°.

You Do 3
1. Reflex $\angle AOC = 2 \times 118° = 236°$ *(the angle at the centre is twice the angle at the circumference)*.
2. Angles around a point sum to $360°$:
Obtuse $\angle AOC = 360° - 236° = 124°$.
3. $OA = OC$ *(radii of the circle)*, so triangle OAC is isosceles:
$\angle OAC = (180° - 124°) \div 2 = 56° \div 2 =$ **28**°.

Challenge 1

1. Since M is the midpoint of chord AB, $\angle OMA = 90°$ *(the perpendicular from the centre of a circle to a chord bisects the chord)*, and $AM = 16 \div 2 = 8$ cm.
2. Use Pythagoras' theorem in right-angled triangle OMA:
$OM^2 + 8^2 = 10^2 \implies OM^2 = 100 - 64 = 36 \implies OM = 6$ cm.
3. Since OC is a radius of length 10 cm:
$MC = OC - OM = 10 - 6 = 4$ cm.
4. Use Pythagoras' theorem in right-angled triangle AMC:
$AC^2 = AM^2 + MC^2 = 8^2 + 4^2 = 64 + 16 = 80$.
$AC = \sqrt{80} = \sqrt{16 \times 5} = \mathbf{4\sqrt{5}\,cm}$.

Challenge 2

1. Use the angle at the centre theorem:
$\angle ADC = \frac{1}{2}\angle AOC = \frac{2x+40}{2} = (x+20)°$ *(the angle at the centre is twice the angle at the circumference)*.
2. $ABCD$ is a cyclic quadrilateral *(opposite angles in a cyclic quadrilateral add up to $180°$)*:
$\angle ADC + \angle ABC = 180°$
$(x + 20) + (3x - 20) = 180$
$4x = 180 \implies \mathbf{x = 45}$.
3. Substitute $x = 45$ to find angle ADC:
$\angle ADC = 45 + 20 = \mathbf{65}°$.

Extreme Challenge

1. Draw the radius OC.
$\angle OAP = 90°$ and $\angle OCP = 90°$ *(a tangent is perpendicular to the radius at the point of contact)*.
2. In triangles OAP and OCP:
- $\angle OAP = \angle OCP = 90°$
- OP is a common hypotenuse
- $OA = OC$ *(radii of the circle)*

Therefore, $\triangle OAP$ and $\triangle OCP$ are congruent *(RHS)*.
3. By congruence, OP bisects $\angle APC$, so $\angle APO = x$.
In right-angled triangle OAP, angles sum to $180°$:
$\angle AOP = 180° - 90° - x = 90° - x$.
Similarly, $\angle COP = 90° - x$, so $\angle AOC = 2(90° - x) = 180° - 2x$.
4. Use the angle at the centre theorem on arc AC:
$\angle ABC = \frac{1}{2}\angle AOC = \frac{180°-2x}{2} = 90° - x$ *(the angle at the centre is twice the angle at the circumference)*.
5. Since AOB is a straight line (diameter) and $\angle AOP = \angle ABC = 90° - x$, these are equal corresponding angles.
Therefore, **OP is parallel to BC**. *(Proved)*