Laws of Indices

Practice Worksheet (Problem Solving)

I Do

(a) Work out the value of 7^0.

..... *(1 mark)*

(b) p and q are non-zero.
Simplify fully

$$\frac{(p^3q^2)^2}{p^2q^5}$$

..... *(2 marks)*

(Total for Question is 3 marks)

We Do

(a) Work out the value of 12^0.

..... *(1 mark)*

(b) a and b are non-zero.
Simplify fully

$$\frac{(a^4b^3)^2}{a^5b^2}$$

..... *(2 marks)*

(Total for Question is 3 marks)

You Do 1

(a) Work out the value of 5^{-2}.
Give your answer as a fraction.

..... *(1 mark)*

(b) x and y are non-zero.
Simplify fully

$$\frac{(2x^3y)^3}{4x^4y^5}$$

..... *(2 marks)*

(Total for Question is 3 marks)

You Do 2

(a) Work out the value of $27^{\frac{2}{3}}$.

..... *(2 marks)*

(b) m and n are non-zero.
Simplify fully

$$(3m^2n^{-3})^2 \times 2m^{-1}n^4$$

..... *(2 marks)*

(Total for Question is 4 marks)

You Do 3

(a) Work out the value of $\left(\frac{16}{81}\right)^{-\frac{1}{4}}$.

..... *(2 marks)*

(b) c and d are non-zero.
Simplify fully

$$\left(\frac{27c^6}{d^9}\right)^{\frac{1}{3}}$$

..... *(2 marks)*

(Total for Question is 4 marks)

Challenge 1

Given that

$$2^{2x-1} = \frac{8}{\sqrt{2}}$$

find the exact value of x.
Show all of your algebraic working clearly.

$x =$ .....

(Total for Question is 3 marks)

Challenge 2

x and y are non-zero, and

$$\frac{(3x^a y^4)^2}{x^5 y^b} = 9x^3 y^2$$

(a) Find the value of a and the value of b.

$a =$ ..... $b =$ ..... *(2 marks)*

(b) Hence, work out the exact value of $\left(\frac{a}{b}\right)^{-2}$.
Give your answer as an improper fraction in its simplest form.

..... *(2 marks)*

(Total for Question is 4 marks)

Extreme Challenge

Given that

$$9^x = 27^y \qquad \text{and} \qquad \frac{2^{x+1}}{4^y} = \frac{1}{8}$$

find the value of x and the value of y.
You must show all of your algebraic working clearly.

$x =$ ..... $y =$ .....

(Total for Question is 5 marks)

Worked Answers

I Do

(a) Any non-zero number raised to the power of 0 equals 1: **1**.
(b) Expand the power in the numerator: $(p^3q^2)^2 = p^6q^4$.
Divide by p^2q^5 (subtracting powers): $\frac{p^6q^4}{p^2q^5} = p^{6-2}q^{4-5} = \mathbf{\frac{p^4}{q}}$ *(or* $\mathbf{p^4q^{-1}}$*)*.

We Do

(a) $12^0 = \mathbf{1}$.
(b) $\frac{(a^4b^3)^2}{a^5b^2} = \frac{a^8b^6}{a^5b^2} = a^{8-5}b^{6-2} = \mathbf{a^3b^4}$.

You Do 1

(a) $5^{-2} = \frac{1}{5^2} = \mathbf{\frac{1}{25}}$.
(b) Cube every factor in the numerator: $(2x^3y)^3 = 2^3x^9y^3 = 8x^9y^3$.
Divide by $4x^4y^5$: $\frac{8x^9y^3}{4x^4y^5} = 2x^{9-4}y^{3-5} = \mathbf{\frac{2x^5}{y^2}}$ *(or* $\mathbf{2x^5y^{-2}}$*)*.

You Do 2

(a) $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = \mathbf{9}$.
(b) Expand the square: $(3m^2n^{-3})^2 = 9m^4n^{-6}$.
Multiply by $2m^{-1}n^4$: $(9 \times 2)m^{4+(-1)}n^{-6+4} = 18m^3n^{-2} = \mathbf{\frac{18m^3}{n^2}}$.

You Do 3

(a) Apply the negative index by taking the reciprocal, then take the fourth root:
$\left(\frac{16}{81}\right)^{-\frac{1}{4}} = \left(\frac{81}{16}\right)^{\frac{1}{4}} = \frac{\sqrt[4]{81}}{\sqrt[4]{16}} = \mathbf{\frac{3}{2}}$ *(or* **1.5***)*.
(b) Raise each part to the power $\frac{1}{3}$ (cube root):
$\frac{27^{\frac{1}{3}}(c^6)^{\frac{1}{3}}}{(d^9)^{\frac{1}{3}}} = \mathbf{\frac{3c^2}{d^3}}$ *(or* $\mathbf{3c^2d^{-3}}$*)*.

Challenge 1

1. Express the right-hand side as a single power of 2:
$8 = 2^3$ and $\sqrt{2} = 2^{\frac{1}{2}}$.
$\frac{8}{\sqrt{2}} = \frac{2^3}{2^{\frac{1}{2}}} = 2^{3-\frac{1}{2}} = 2^{\frac{5}{2}}$.
2. Equate the powers of 2:
$2x - 1 = \frac{5}{2}$
$2x = \frac{7}{2} \implies \mathbf{x = \frac{7}{4}}$ *(or* **1.75***)*.

Challenge 2

(a) Simplify the left-hand side:

$\frac{(3x^a y^4)^2}{x^5 y^b} = \frac{9x^{2a} y^8}{x^5 y^b} = 9x^{2a-5} y^{8-b}$.

Compare powers with $9x^3y^2$:

For x: $2a - 5 = 3 \implies 2a = 8 \implies$ **a** $= 4$.

For y: $8 - b = 2 \implies$ **b** $= 6$.

(b) Substitute $a = 4$ and $b = 6$:

$\left(\frac{4}{6}\right)^{-2} = \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{\mathbf{9}}{\mathbf{4}}$.

Extreme Challenge

1. Rewrite the first equation using base 3:

$9^x = 27^y \implies (3^2)^x = (3^3)^y \implies 3^{2x} = 3^{3y}$.

Equating powers gives: $2x = 3y$ — (Equation 1).

2. Rewrite the second equation using base 2:

$\frac{2^{x+1}}{(2^2)^y} = 2^{-3} \implies \frac{2^{x+1}}{2^{2y}} = 2^{-3} \implies 2^{x+1-2y} = 2^{-3}$.

Equating powers gives: $x + 1 - 2y = -3 \implies x - 2y = -4 \implies x = 2y - 4$ — (Equation 2).

3. Substitute Equation 2 into Equation 1:

$2(2y - 4) = 3y$

$4y - 8 = 3y \implies$ **y** $= 8$.

4. Substitute $y = 8$ back into Equation 2:

$x = 2(8) - 4 = 16 - 4 =$ **12**.