Recurring Decimals and Fractions

Practice Worksheet (Problem Solving)

I Do

x and y are recurring decimals.

$x = 0.\dot{2} \quad y = 0.6\dot{8}\dot{1}$

Use algebra to show that $xy = \frac{5}{33}$

. . . . . .

(Total for Question is 4 marks)

We Do

***a* and *b* are recurring decimals.**

$a = 0.\dot{4} \quad b = 0.5\dot{3}\dot{6}$

Use algebra to show that $ab = \frac{118}{495}$

. . . . . .

(Total for Question is 4 marks)

You Do 1

***p* and *q* are recurring decimals.**

$p = 0.\dot{5} \quad q = 0.1\dot{2}\dot{7}$

Use algebra to show that $pq = \frac{7}{99}$

. . . . . .

(Total for Question is 4 marks)

You Do 2

***u* and *v* are recurring decimals.**

$u = 0.\dot{1}\dot{5} \quad v = 0.3\dot{8}$

Use algebra to show that $\frac{u}{v} = \frac{30}{77}$

. . . . . .

(Total for Question is 4 marks)

You Do 3

m **and** ***n*** **are recurring decimals.**

$m = 1.\dot{6} \quad n = 0.2\dot{4}$

Use algebra to show that $m - n = \frac{64}{45}$

. . . . . .

(Total for Question is 4 marks)

Challenge 1

***x* and *y* are recurring decimals.**

$x = 0.\dot{7} \quad y = 0.\dot{3}\dot{6}$

Use algebra to show that $\frac{x+y}{xy} = \frac{113}{28}$

. . . . . .

(Total for Question is 5 marks)

Challenge 2

Use algebra to show that the exact solution to the equation

$$0.\dot{1}\dot{8} \times w = 0.4\dot{7}$$

is $w = \frac{473}{180}$

. . . . . .

(Total for Question is 4 marks)

Extreme Challenge

The number r is given by

$$r = \sqrt{0.\dot{4} \times 0.02\dot{7}}$$

Use algebra to show that $r = \frac{1}{9}$

. . . . . .

(Total for Question is 5 marks)

Worked Answers

I Do

First, convert both decimals to fractions.
$x = 0.\dot{2} = \frac{2}{9}$
Let $y = 0.6818181\ldots$
$10y = 6.818181\ldots$
$1000y = 681.818181\ldots$
$1000y - 10y = 681.8181\cdots - 6.8181\ldots$
$990y = 675 \implies y = \frac{675}{990}$
Simplify y: divide numerator and denominator by 45 (or repeatedly by 5 and 9).
$y = \frac{15}{22}$
Now, multiply x and y:
$xy = \frac{2}{9} \times \frac{15}{22} = \frac{30}{198}$
Simplify by dividing by 6: $\frac{30\div6}{198\div6} = \mathbf{\frac{5}{33}}$.

We Do

$a = 0.\dot{4} = \frac{4}{9}$
Let $b = 0.5363636\ldots$
$10b = 5.363636\ldots$
$1000b = 536.363636\ldots$
$990b = 536 - 5 = 531 \implies b = \frac{531}{990}$
Simplify b: divide by 9 to get $\frac{59}{110}$.
Multiply a and b:
$ab = \frac{4}{9} \times \frac{59}{110} = \frac{236}{990}$
Simplify by dividing by 2: $\mathbf{\frac{118}{495}}$.

You Do 1

$p = 0.\dot{5} = \frac{5}{9}$
Let $q = 0.1272727\ldots$
$10q = 1.272727\ldots$
$1000q = 127.272727\ldots$
$990q = 127 - 1 = 126 \implies q = \frac{126}{990}$
Simplify q: divide by 18 to get $\frac{7}{55}$.
Multiply p and q:
$pq = \frac{5}{9} \times \frac{7}{55} = \frac{35}{495}$
Simplify by dividing by 5: $\mathbf{\frac{7}{99}}$.

You Do 2
Let $u = 0.\dot{1}\dot{5} = \frac{15}{99} = \frac{5}{33}$.
Let $v = 0.3\dot{8} = 0.3888\ldots$
$10v = 3.888\ldots$
$100v = 38.888\ldots$
$90v = 35 \implies v = \frac{35}{90} = \frac{7}{18}$.
Calculate $\frac{u}{v}$ by multiplying by the reciprocal:
$\frac{u}{v} = \frac{5}{33} \div \frac{7}{18} = \frac{5}{33} \times \frac{18}{7}$
Cross-simplify the 18 and 33 by dividing by 3:
$\frac{5}{11} \times \frac{6}{7} = \mathbf{\frac{30}{77}}$.

You Do 3
Let $m = 1.\dot{6} = 1\frac{6}{9} = \frac{15}{9} = \frac{5}{3}$.
Let $n = 0.2\dot{4} = 0.2444\ldots$
$90n = 24 - 2 = 22 \implies n = \frac{22}{90} = \frac{11}{45}$.
Calculate $m - n$:
$m = \frac{5}{3} = \frac{75}{45}$
$m - n = \frac{75}{45} - \frac{11}{45} = \mathbf{\frac{64}{45}}$ (or $1\frac{19}{45}$).

Challenge 1
$x = 0.\dot{7} = \frac{7}{9}$
$y = 0.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$
Numerator $(x + y)$: $\frac{7}{9} + \frac{4}{11} = \frac{77}{99} + \frac{36}{99} = \frac{113}{99}$.
Denominator (xy): $\frac{7}{9} \times \frac{4}{11} = \frac{28}{99}$.
Divide numerator by denominator:
$\frac{113}{99} \div \frac{28}{99} = \frac{113}{99} \times \frac{99}{28} = \mathbf{\frac{113}{28}}$.

Challenge 2
Convert both to fractions:
$0.\dot{1}\dot{8} = \frac{18}{99} = \frac{2}{11}$.
$0.4\dot{7} = \frac{47-4}{90} = \frac{43}{90}$.
Equation becomes: $\frac{2}{11}w = \frac{43}{90}$.
Multiply by $\frac{11}{2}$ to isolate w:
$w = \frac{43}{90} \times \frac{11}{2} = \mathbf{\frac{473}{180}}$.

Extreme Challenge
Convert the decimals to fractions:
$0.\dot{4} = \frac{4}{9}$.
For $0.02\dot{7}$:
Let $z = 0.02777\ldots$
$100z = 2.777\ldots$
$1000z = 27.777\ldots$
$900z = 25 \implies z = \frac{25}{900}$.
Simplify z: divide by 25 $\implies z = \frac{1}{36}$.
Now multiply the fractions inside the square root:
$0.\dot{4} \times 0.02\dot{7} = \frac{4}{9} \times \frac{1}{36} = \frac{4}{324}$.
Simplify the fraction: $\frac{4}{324} = \frac{1}{81}$.

Calculate the square root: $r = \sqrt{\frac{1}{81}} = \mathbf{\frac{1}{9}}$.