Adding & Subtracting Algebraic Fractions

Practice Worksheet (Problem Solving)

I Do

Express as a single fraction in its simplest form.

$$\frac{3}{x+2} - \frac{1}{x-5}$$

(Total for Question is 3 marks)

We Do

Express as a single fraction in its simplest form.

$$\frac{4}{x-3} + \frac{2}{x+1}$$

(Total for Question is 3 marks)

You Do 1

Express as a single fraction in its simplest form.

$$\frac{5}{x+4} - \frac{3}{x-2}$$

(Total for Question is 3 marks)

Video walkthrough

You Do 2

Express as a single fraction in its simplest form.

$$\frac{2}{2x+1} + \frac{3}{x-4}$$

(Total for Question is 3 marks)

You Do 3

Express as a single fraction in its simplest form.

$$\frac{x}{x+3} - \frac{4}{x-1}$$

(Total for Question is 3 marks)

Challenge 1

Express as a single fraction in its simplest form.

$$\frac{4}{x^2-25} + \frac{3}{x-5}$$

Show clear algebraic working.

(Total for Question is 4 marks)

Challenge 2

Solve the equation

$$\frac{3}{x-1} + \frac{2}{x+1} = 2$$

Show clear algebraic working.

$x =$ .....

(Total for Question is 5 marks)

Extreme Challenge

Express as a single fraction in its simplest form.

$$\frac{2x}{x^2+5x+6} - \frac{3}{x^2+x-2}$$

You must show all your algebraic working clearly.

(Total for Question is 5 marks)

Answers & Worked Solutions

I Do

Find a common denominator: $(x+2)(x-5)$.
Multiply the numerator and denominator of the first fraction by $(x-5)$:
$\frac{3(x-5)}{(x+2)(x-5)}$
Multiply the numerator and denominator of the second fraction by $(x+2)$:
$\frac{1(x+2)}{(x+2)(x-5)}$
Subtract the numerators over the common denominator:
$\frac{3(x-5)-1(x+2)}{(x+2)(x-5)}$
Expand the brackets on the numerator carefully:
$\frac{3x-15-x-2}{(x+2)(x-5)}$
Collect like terms:
$\frac{\mathbf{2x-17}}{\mathbf{(x+2)(x-5)}}$

We Do

Common denominator: $(x-3)(x+1)$.
Cross-multiply to adjust numerators:
$\frac{4(x+1)}{(x-3)(x+1)} + \frac{2(x-3)}{(x-3)(x+1)}$
Add the numerators:
$\frac{4(x+1)+2(x-3)}{(x-3)(x+1)}$
Expand brackets:
$\frac{4x+4+2x-6}{(x-3)(x+1)}$
Collect like terms:
$\frac{\mathbf{6x-2}}{\mathbf{(x-3)(x+1)}}$

You Do 1

Common denominator: $(x+4)(x-2)$.
$\frac{5(x-2)}{(x+4)(x-2)} - \frac{3(x+4)}{(x+4)(x-2)}$
Combine into a single fraction:
$\frac{5(x-2)-3(x+4)}{(x+4)(x-2)}$
Expand brackets (be careful with the -3):
$\frac{5x-10-3x-12}{(x+4)(x-2)}$
Collect like terms:
$\frac{\mathbf{2x-22}}{\mathbf{(x+4)(x-2)}}$

You Do 2

Common denominator: $(2x+1)(x-4)$.
$\frac{2(x-4)}{(2x+1)(x-4)} + \frac{3(2x+1)}{(2x+1)(x-4)}$
Combine into a single fraction:
$\frac{2(x-4)+3(2x+1)}{(2x+1)(x-4)}$
Expand brackets:
$\frac{2x-8+6x+3}{(2x+1)(x-4)}$
Collect like terms:
$\frac{\mathbf{8x-5}}{\mathbf{(2x+1)(x-4)}}$

You Do 3

Common denominator: $(x+3)(x-1)$.

$\frac{x(x-1)}{(x+3)(x-1)} - \frac{4(x+3)}{(x+3)(x-1)}$

Combine into a single fraction:

$\frac{x(x-1)-4(x+3)}{(x+3)(x-1)}$

Expand brackets:

$\frac{x^2-x-4x-12}{(x+3)(x-1)}$

Collect like terms:

$\frac{\mathbf{x^2-5x-12}}{\mathbf{(x+3)(x-1)}}$

Challenge 1

Notice that the denominator of the first fraction is a Difference of Two Squares:

$x^2 - 25 = (x-5)(x+5)$

The expression is:

$\frac{4}{(x-5)(x+5)} + \frac{3}{x-5}$

The lowest common denominator is simply $(x-5)(x+5)$.

The first fraction does not need changing.

Multiply the numerator and denominator of the second fraction by $(x+5)$:

$\frac{4}{(x-5)(x+5)} + \frac{3(x+5)}{(x-5)(x+5)}$

Add the numerators:

$\frac{4+3(x+5)}{(x-5)(x+5)}$

Expand and simplify:

$\frac{4+3x+15}{(x-5)(x+5)} = \frac{\mathbf{3x+19}}{\mathbf{(x-5)(x+5)}}$

Challenge 2

Combine the left side into a single fraction:

Common denominator is $(x-1)(x+1)$.

$\frac{3(x+1)+2(x-1)}{(x-1)(x+1)} = 2$

Simplify the numerator:

$\frac{3x+3+2x-2}{(x-1)(x+1)} = 2$

$\frac{5x+1}{(x-1)(x+1)} = 2$

Multiply both sides by the denominator, $(x-1)(x+1)$:

$5x + 1 = 2(x-1)(x+1)$

Expand the difference of two squares:

$5x + 1 = 2(x^2 - 1)$

$5x + 1 = 2x^2 - 2$

Rearrange to form a quadratic equal to zero:

$0 = 2x^2 - 5x - 3$

Factorise the quadratic:

$0 = (2x+1)(x-3)$

Solve for x:

$2x + 1 = 0 \implies 2x = -1 \implies x = -0.5$

$x - 3 = 0 \implies x = 3$

$\mathbf{x = 3}$ or $\mathbf{x = -0.5}$

Extreme Challenge

First, fully factorise the quadratic denominators:

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$x^2 + x - 2 = (x + 2)(x - 1)$

Rewrite the expression:

$\frac{2x}{(x+2)(x+3)} - \frac{3}{(x+2)(x-1)}$

Find the lowest common denominator (LCD), which is $(x + 2)(x + 3)(x - 1)$.

Multiply the first fraction by $\frac{x-1}{x-1}$:

$\frac{2x(x-1)}{(x+2)(x+3)(x-1)}$

Multiply the second fraction by $\frac{x+3}{x+3}$:

$\frac{3(x+3)}{(x+2)(x+3)(x-1)}$

Subtract the numerators:

$\frac{2x(x-1)-3(x+3)}{(x+2)(x+3)(x-1)}$

Expand brackets:

$\frac{2x^2-2x-3x-9}{(x+2)(x+3)(x-1)}$

Collect like terms:

$\frac{\mathbf{2x^2-5x-9}}{\mathbf{(x+2)(x+3)(x-1)}}$