Circle Theorems

Practice Worksheet (Problem Solving)

I Do

A, B, C and D lie on the circumference of a circle with centre O.
Angle AOC = 130°. B and D lie on opposite arcs AC.

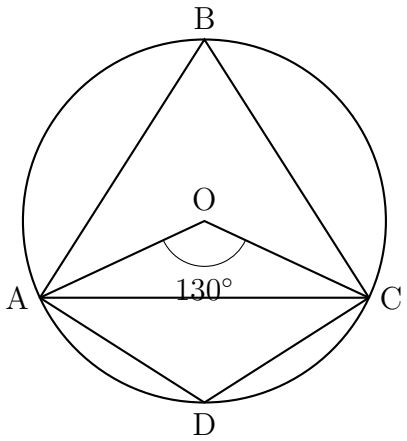


Diagram not to scale

(a) Work out angle ABC. Give a reason for your answer.

(2)

(b) Work out angle ADC. Give a reason for your answer.

(2)

(Total for Question is 4 marks)

Video walkthrough

We Do

P, Q, R and S lie on the circumference of a circle with centre O.
Angle POR = 110°. Q and S lie on opposite arcs PR.

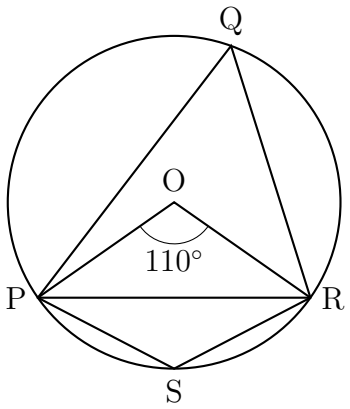


Diagram not to scale

(a) Work out angle PQR. Give a reason for your answer.

(2)

(b) Work out angle PSR. Give a reason for your answer.

(2)

(Total for Question is 4 marks)

You Do 1

W, X, Y and Z lie on the circumference of a circle with centre O.
Angle WOY = 144°. X and Z lie on opposite arcs WY.

X lies on the major arc WY and Z lies on the minor arc WY.

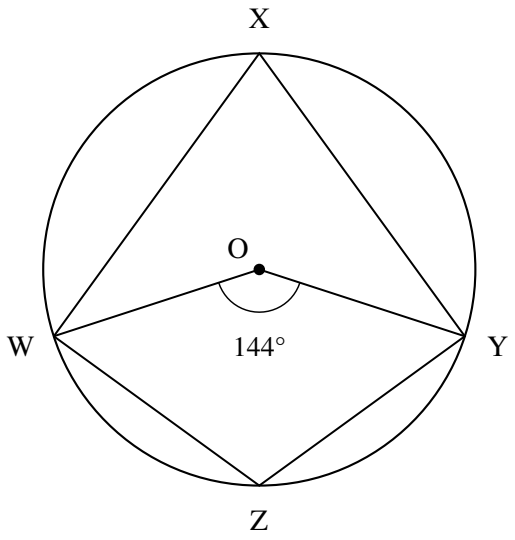


Diagram not to scale

(a) Work out angle WXY. Give a reason for your answer.

(2)

(b) Work out angle WZY. Give a reason for your answer.

(2)

(Total for Question is 4 marks)

You Do 2

A, B, C and D lie on the circumference of a circle with centre O.
Angle ABC = 62°.

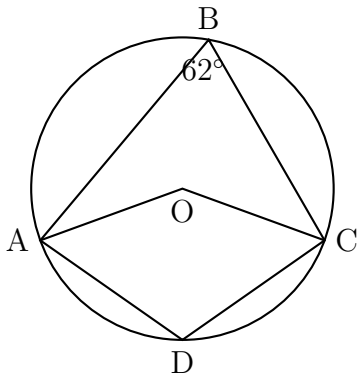


Diagram not to scale

(a) Work out angle AOC. Give a reason for your answer.

(2)

(b) Work out angle ADC. Give a reason for your answer.

(2)

(Total for Question is 4 marks)

You Do 3

E, F, G and H lie on the circumference of a circle with centre O.
Angle EHG = 108°.

E, F, G and H form a cyclic quadrilateral, in that order.

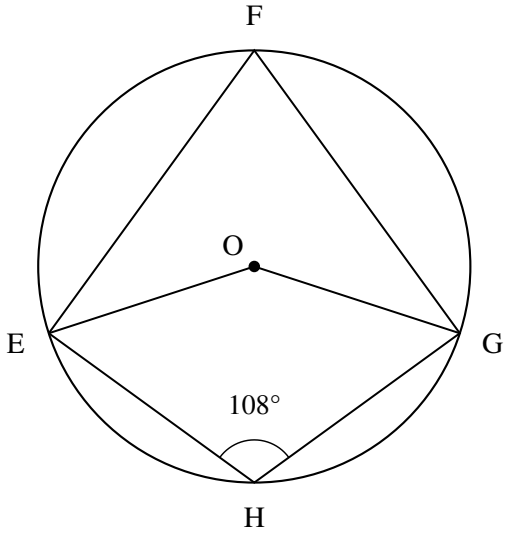


Diagram not to scale

(a) Work out angle EFG.

(1)

(b) Work out the reflex angle EOG. Give a reason for your answer.

(2)

(Total for Question is 3 marks)

Challenge 1

A, B, C and D lie on the circumference of a circle with centre O.
Angle AOC = 128°.

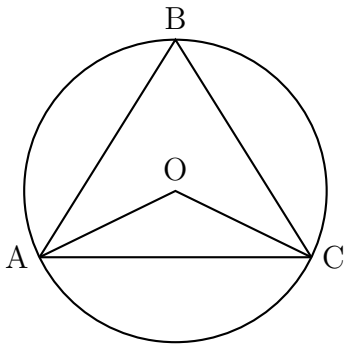


Diagram not to scale

(a) Work out the size of angle OAC.
You must show your working and state any geometric facts you use.

(3)

(b) Work out the size of angle ABC.

(1)

(Total for Question is 4 marks)

Challenge 2

P, Q, R and S form a cyclic quadrilateral inside a circle.
Angle PQR = $(3x + 10)°$.
Angle PSR = $(5x - 30)°$.

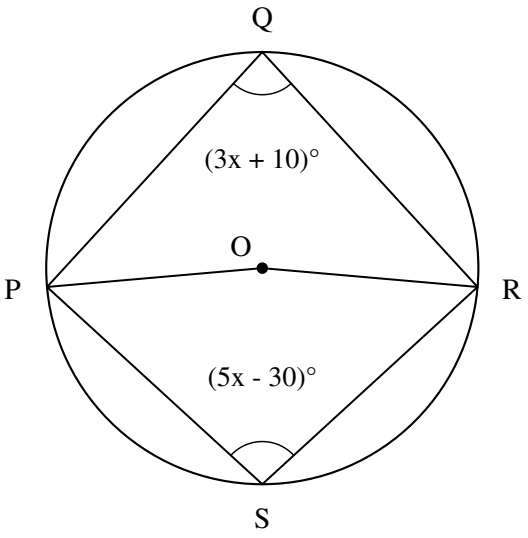


Diagram not to scale

(a) Work out the value of x.
Show clear algebraic working.

$x =$ .....

(3)

(b) The centre of the circle is O.
Find the size of the obtuse angle POR.

(2)

(Total for Question is 5 marks)

Extreme Challenge

A, B, C and D lie on the circumference of a circle with centre O.
Angle ABC and Angle ADC are opposite angles of the cyclic quadrilateral ABCD.

The angle at the centre, AOC, is $(5x + 12)°$.
The angle at the circumference on the major arc, ABC, is $(3x - 4)°$.

Work out the size of angle ADC.
You must show all your algebraic working clearly.

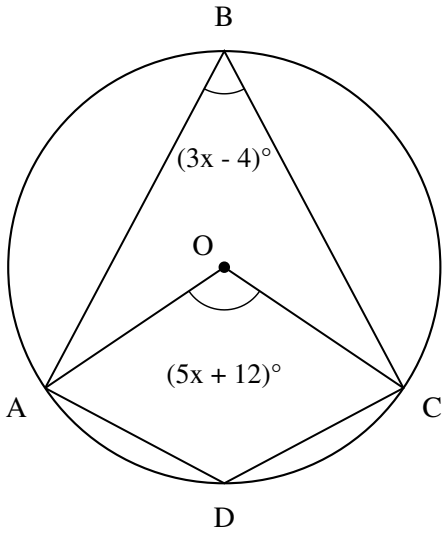


Diagram not to scale

(Total for Question is 5 marks)

Answers & Worked Solutions

I Do

(a) Angle ABC $= 130° \div 2 =$ **65**°.
Reason: The angle at the centre is twice the angle at the circumference (subtended by the same arc).
(b) Angle ADC $= 180° - 65° =$ **115**°.
Reason: Opposite angles in a cyclic quadrilateral sum to 180°.

We Do

(a) Angle PQR $= 110° \div 2 =$ **55**°.
Reason: The angle at the centre is twice the angle at the circumference.
(b) Angle PSR $= 180° - 55° =$ **125**°.
Reason: Opposite angles in a cyclic quadrilateral sum to 180°.

You Do 1

(a) Angle WXY $= 144° \div 2 =$ **72**°.
Reason: Angle at the centre is twice the angle at the circumference.
(b) Angle WZY $= 180° - 72° =$ **108**°.
Reason: Opposite angles in a cyclic quadrilateral sum to 180°.

You Do 2

(a) Angle AOC $= 62° \times 2 =$ **124**°.
Reason: Angle at the centre is twice the angle at the circumference.
(b) Angle ADC $= 180° - 62° =$ **118**°.
Reason: Opposite angles in a cyclic quadrilateral sum to 180°.

You Do 3

(a) Opposite angles of the cyclic quadrilateral sum to 180°.

Angle EFG = 180° − 108° = 72°.

(b) Angle EHG subtends the major arc EFG.

The reflex angle EOG is twice this angle at the circumference:

Reflex angle EOG = 2 × 108° = 216°.

Challenge 1

(a) Triangle AOC is an isosceles triangle because OA and OC are both radii of the circle.
Base angles in an isosceles triangle are equal.
Angle OAC + Angle OCA $= 180° - 128° = 52°$.
Angle OAC $= 52° \div 2 =$ **26**°.
(b) Angle ABC $= 128° \div 2 =$ **64**°.

Challenge 2

(a) Opposite angles in a cyclic quadrilateral sum to 180°.

$(3x + 10) + (5x - 30) = 180$

$8x - 20 = 180$

$8x = 200 \implies x = 200 \div 8 = \mathbf{25}$.

(b) First find the angle PQR at the circumference:

PQR $= 3(25) + 10 = 75 + 10 = 85°$.

The obtuse angle POR at the centre is twice the angle at the circumference (PQR).

Obtuse POR $= 85° \times 2 = \mathbf{170}°$.

Extreme Challenge

The angle at the centre is twice the angle at the circumference.

$5x + 12 = 2(3x - 4)$

$5x + 12 = 6x - 8$

Subtract $5x$ from both sides: $12 = x - 8$

Add 8 to both sides: $x = 20$.

Now substitute x back in to find angle ABC:

Angle ABC $= 3(20) - 4 = 60 - 4 = 56°$.

ABCD forms a cyclic quadrilateral, so opposite angles sum to 180°.

Angle ADC $= 180° - 56° = \mathbf{124}°$.