Conditional Probability

Practice Worksheet (Problem Solving)

I Do

There are 7 red counters and 5 blue counters in a bag.
Emma takes a counter at random, keeps it, and then takes a second counter.
Work out the probability that Emma takes counters of different colours.

(Total for Question is 4 marks)

We Do

There are 8 green counters and 4 yellow counters in a bag.
Tom takes a counter at random, keeps it, and then takes a second counter.
Work out the probability that Tom takes counters of different colours.

(Total for Question is 4 marks)

You Do 1

There are 6 black counters and 4 white counters in a bag.
Catherine takes a counter at random from the bag and does not replace it.
She then takes a second counter.
Work out the probability that Catherine takes two counters of the same colour.

(Total for Question is 4 marks)

Video walkthrough

You Do 2

There are 5 red counters, 4 blue counters, and 3 green counters in a bag.
John takes two counters at random from the bag without replacement.
Work out the probability that both counters are the same colour.

(Total for Question is 4 marks)

You Do 3

There are 4 orange counters, 5 purple counters, and 2 pink counters in a bag.
Ryan takes two counters at random from the bag, one after the other, without replacement.
Work out the probability that neither counter is pink.

(Total for Question is 3 marks)

Challenge 1

There are x red counters and 4 blue counters in a bag.
Paul takes two counters at random from the bag, without replacement.
The probability that Paul takes two red counters is $\frac{1}{3}$.

Work out the value of x.
Show clear algebraic working.

$x =$ .....

(Total for Question is 5 marks)

Challenge 2

There are 6 red counters and 4 blue counters in a bag.
Sophie takes three counters at random from the bag, without replacement.
Work out the probability that Sophie takes exactly two red counters and one blue counter.

(Total for Question is 4 marks)

Extreme Challenge

There are x red counters and $(x + 2)$ blue counters in a bag.
Paige takes two counters at random from the bag, without replacement.
The probability that she takes counters of different colours is $\frac{35}{66}$.

Find the total number of counters originally in the bag.
Show clear algebraic working.

(Total for Question is 6 marks)

Answers & Worked Solutions

I Do
Total number of counters $= 7 + 5 = 12$.
Emma can pick different colours in two ways: Red then Blue (RB) OR Blue then Red (BR).
$P(RB) = \frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$
$P(BR) = \frac{5}{12} \times \frac{7}{11} = \frac{35}{132}$
Total probability $= P(RB) + P(BR) = \frac{35}{132} + \frac{35}{132} = \frac{70}{132}$.
Simplify the fraction by dividing by 2: $\mathbf{\frac{35}{66}}$.

We Do
Total number of counters $= 8 + 4 = 12$.
Different colours: Green then Yellow (GY) OR Yellow then Green (YG).
$P(GY) = \frac{8}{12} \times \frac{4}{11} = \frac{32}{132}$
$P(YG) = \frac{4}{12} \times \frac{8}{11} = \frac{32}{132}$
Total probability $= \frac{32}{132} + \frac{32}{132} = \frac{64}{132}$.
Simplify the fraction by dividing by 4: $\mathbf{\frac{16}{33}}$.

You Do 1
Total number of counters $= 6 + 4 = 10$.
Same colour: Black then Black (BB) OR White then White (WW).
$P(BB) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$
$P(WW) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$
Total probability $= \frac{30}{90} + \frac{12}{90} = \frac{42}{90}$.
Simplify the fraction by dividing by 6: $\mathbf{\frac{7}{15}}$.

You Do 2
Total number of counters $= 5 + 4 + 3 = 12$.
Same colour: Red-Red (RR) OR Blue-Blue (BB) OR Green-Green (GG).
$P(RR) = \frac{5}{12} \times \frac{4}{11} = \frac{20}{132}$
$P(BB) = \frac{4}{12} \times \frac{3}{11} = \frac{12}{132}$
$P(GG) = \frac{3}{12} \times \frac{2}{11} = \frac{6}{132}$
Total probability $= \frac{20}{132} + \frac{12}{132} + \frac{6}{132} = \frac{38}{132}$.
Simplify by dividing by 2: $\mathbf{\frac{19}{66}}$.

You Do 3
Total number of counters $= 4 + 5 + 2 = 11$.
If neither counter is pink, Ryan must be picking from the Non-Pink counters.
Number of Non-Pink counters $= 4$ (orange) $+ 5$ (purple) $= 9$.
$P(\text{Not Pink, Not Pink}) = \frac{9}{11} \times \frac{8}{10} = \frac{72}{110}$.
Simplify by dividing by 2: $\mathbf{\frac{36}{55}}$.

Challenge 1

Total number of counters initially $= x + 4$.

$P(\text{Red then Red}) = \left(\frac{x}{x+4}\right) \times \left(\frac{x-1}{x+3}\right) = \frac{1}{3}$.

Multiply the numerators and denominators:

$\frac{x(x-1)}{(x+4)(x+3)} = \frac{1}{3}$

Cross-multiply to remove fractions:

$3x(x-1) = 1(x+4)(x+3)$

$3x^2 - 3x = x^2 + 7x + 12$

Subtract $x^2 + 7x + 12$ from both sides to form a quadratic equal to zero:

$2x^2 - 10x - 12 = 0$

Divide the entire equation by 2:

$x^2 - 5x - 6 = 0$

Factorise:

$(x-6)(x+1) = 0$

Since the number of counters cannot be negative, $x = \mathbf{6}$.

Challenge 2

Total number of counters $= 10$.

Three combinations give exactly two reds and one blue: RRB, RBR, and BRR.

$P(RRB) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{120}{720} = \frac{1}{6}$

$P(RBR) = \frac{6}{10} \times \frac{4}{9} \times \frac{5}{8} = \frac{120}{720} = \frac{1}{6}$

$P(BRR) = \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{120}{720} = \frac{1}{6}$

Total probability $= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \mathbf{\frac{1}{2}}$.

Extreme Challenge

Total counters $= x + (x+2) = 2x + 2$.

Different colours means Red-Blue (RB) OR Blue-Red (BR).

$P(RB) = \left(\frac{x}{2x+2}\right) \times \left(\frac{x+2}{2x+1}\right) = \frac{x(x+2)}{(2x+2)(2x+1)}$

$P(BR)$ is identical to $P(RB)$, so Total Probability $= 2 \times P(RB)$.

$2 \times \frac{x(x+2)}{2(x+1)(2x+1)} = \frac{35}{66}$

Notice the 2 on the top and the 2 factored out of $(2x+2)$ cancel each other out:

$\frac{x(x+2)}{(x+1)(2x+1)} = \frac{35}{66}$

Expand the brackets on the bottom:

$\frac{x^2+2x}{2x^2+3x+1} = \frac{35}{66}$

Cross-multiply:

$66(x^2 + 2x) = 35(2x^2 + 3x + 1)$

$66x^2 + 132x = 70x^2 + 105x + 35$

Rearrange to form a quadratic equal to zero (move everything to the right):

$0 = 4x^2 - 27x + 35$

Factorise the quadratic:

$0 = (4x - 7)(x - 5)$

This gives $x = \frac{7}{4}$ (reject, must be an integer) or $x = 5$.

If $x = 5$, there are 5 Red and 7 Blue counters.

Total counters $= 5 + 7 = \mathbf{12}$.