# Cone Surface Area & Pythagoras

*Practice Worksheet (Problem Solving)*

**Exam Formula:**
The curved surface area of a cone with base radius $r$ and slant height $l$ is $\pi\mathbf{rl}$.

## I Do

A solid cone has base radius $r$ cm, perpendicular height $h$ cm and slant height $l$ cm.
Its total surface area is $24\pi$ cm$^2$.
Given that $l = 3r$, find the exact value of $h$.
Give your answer in the form $a\sqrt{b}$, where $a$ and $b$ are integers.

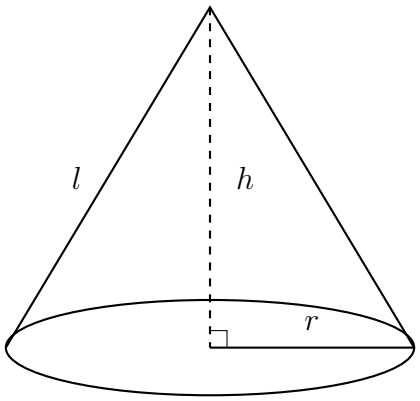


Diagram not to scale

**(Total for Question is 4 marks)**

Video walkthrough

# We Do

**Exam Formula:**
The curved surface area of a cone with base radius $r$ and slant height $l$ is $\pi\mathbf{rl}$.

A solid cone has base radius $r$ cm, perpendicular height $h$ cm and slant height $l$ cm.
Its total surface area is $90\pi$ cm$^2$.
Given that $l = 4r$, find the exact value of $h$.
Give your answer in the form $a\sqrt{b}$, where $a$ and $b$ are integers.

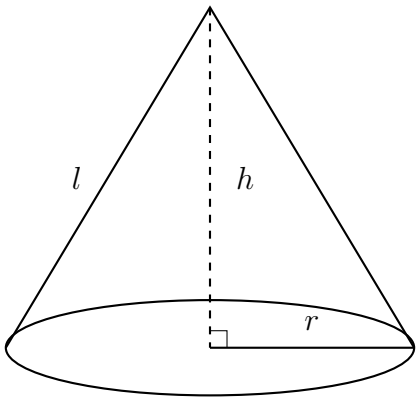


Diagram not to scale

**(Total for Question is 4 marks)**

# You Do 1

A solid cone has base radius $r$ cm, perpendicular height $h$ cm and slant height $l$ cm.
Its total surface area is $48\pi$ cm$^2$.
Given that $l = 2r$, find the exact value of $h$.
Give your answer in the form $a\sqrt{b}$, where $a$ and $b$ are integers.

**(Total for Question is 4 marks)**

# You Do 2

A solid cone has base radius $r$ cm, perpendicular height $h$ cm and slant height $l$ cm.
Its total surface area is $144\pi$ cm$^2$.
Given that $l = 3r$, find the exact value of $h$.
Give your answer in the form $a\sqrt{b}$, where $a$ and $b$ are integers.

**(Total for Question is 4 marks)**

# You Do 3

A solid cone has base radius $r$ cm, perpendicular height $h$ cm and slant height $l$ cm.
Its total surface area is $96\pi$ cm$^2$.
Given that $l = 5r$, find the exact value of $h$.
Give your answer in the form $a\sqrt{b}$, where $a$ and $b$ are integers.

**(Total for Question is 4 marks)**

# Challenge 1

**Exam Formula:**
The volume of a cone with base radius $r$ and perpendicular height $h$ is $\frac{1}{3}\pi\mathbf{r^2h}$.

A solid cone has base radius $r$, perpendicular height $h$, and slant height $l$.
The volume of the cone is $18\pi$ cm$^3$.
Given that $h = 2r$, work out the exact total surface area of the cone.
Give your answer in the form $a\pi(1 + \sqrt{b})$, where $a$ and $b$ are integers.

**(Total for Question is 5 marks)**

# Challenge 2

**Exam Formula:**
The surface area of a sphere with radius $r$ is $\mathbf{4\pi r^2}$.

A solid cone and a solid hemisphere share the same base radius, $r$.
The cone has a perpendicular height $h$ and a slant height $l$.

The total surface area of the solid cone is exactly equal to the total surface area of the solid hemisphere.

Prove that $h = r\sqrt{3}$.
You must show all your algebraic working clearly.

**(Total for Question is 5 marks)**

# Extreme Challenge

A solid cone has a curved surface area of $A$ and a circular base area of $B$.
It is given that $A = 3B$.

Prove that the exact volume of the cone can be written in the form

$$\frac{2}{3}B\sqrt{\frac{2B}{\pi}}$$

You must show all your algebraic steps clearly.

**(Total for Question is 6 marks)**

## Answers & Worked Solutions

### I Do

The total surface area is the base area plus the curved surface area: TSA $= \pi r^2 + \pi rl$.
We are given that TSA $= 24\pi$ and $l = 3r$.
Substitute $l = 3r$ into the formula:
$\pi r^2 + \pi r(3r) = 24\pi$
Divide the entire equation by $\pi$:
$r^2 + 3r^2 = 24$
$4r^2 = 24 \implies r^2 = 6$.
Since $r$ is a length, $r = \sqrt{6}$.
The slant height $l = 3r = 3\sqrt{6}$.
Use Pythagoras' theorem on the right-angled triangle formed by $r, h$, and $l$:
$h^2 + r^2 = l^2$
$h^2 + (\sqrt{6})^2 = (3\sqrt{6})^2$
$h^2 + 6 = 9 \times 6$
$h^2 + 6 = 54 \implies h^2 = 48$.
$h = \sqrt{48} = \sqrt{16 \times 3} = \mathbf{4\sqrt{3}}$.

### We Do

TSA $= \pi r^2 + \pi rl = 90\pi$.
Substitute $l = 4r$:
$\pi r^2 + \pi r(4r) = 90\pi$
$r^2 + 4r^2 = 90$
$5r^2 = 90 \implies r^2 = 18 \implies r = \sqrt{18} = 3\sqrt{2}$.
$l = 4r = 4(3\sqrt{2}) = 12\sqrt{2}$.
Using Pythagoras: $h^2 = l^2 - r^2$.
$h^2 = (12\sqrt{2})^2 - (\sqrt{18})^2$
$h^2 = (144 \times 2) - 18 = 288 - 18 = 270$.
$h = \sqrt{270} = \sqrt{9 \times 30} = \mathbf{3\sqrt{30}}$.

### You Do 1

$\pi r^2 + \pi rl = 48\pi$.
Substitute $l = 2r$:
$r^2 + 2r^2 = 48 \implies 3r^2 = 48 \implies r^2 = 16 \implies r = 4$.
$l = 2r = 8$.
$h^2 = l^2 - r^2 = 8^2 - 4^2 = 64 - 16 = 48$.
$h = \sqrt{48} = \sqrt{16 \times 3} = \mathbf{4\sqrt{3}}$.

### You Do 2

$\pi r^2 + \pi r(3r) = 144\pi$
$4r^2 = 144 \implies r^2 = 36 \implies r = 6$.
$l = 3r = 18$.
$h^2 = l^2 - r^2 = 18^2 - 6^2 = 324 - 36 = 288$.
$h = \sqrt{288} = \sqrt{144 \times 2} = \mathbf{12\sqrt{2}}$.

**You Do 3**

$\pi r^2 + \pi r(5r) = 96\pi$
$6r^2 = 96 \implies r^2 = 16 \implies r = 4.$
$l = 5r = 20.$
$h^2 = l^2 - r^2 = 20^2 - 4^2 = 400 - 16 = 384.$
$h = \sqrt{384} = \sqrt{64 \times 6} = \mathbf{8\sqrt{6}}.$

**Challenge 1**

Volume $= \frac{1}{3}\pi r^2 h = 18\pi.$
Substitute $h = 2r$:
$\frac{1}{3}\pi r^2(2r) = 18\pi \implies \frac{2}{3}r^3 = 18$
$r^3 = 18 \times \frac{3}{2} = 27 \implies r = 3.$
If $r = 3$, then $h = 2(3) = 6.$
Find $l$ using Pythagoras: $l^2 = r^2 + h^2 = 3^2 + 6^2 = 9 + 36 = 45.$
$l = \sqrt{45} = 3\sqrt{5}.$
Total Surface Area $= \pi r^2 + \pi rl = \pi(3^2) + \pi(3)(3\sqrt{5}) = 9\pi + 9\pi\sqrt{5}.$
Factorise out $9\pi$: $\mathbf{9\pi(1 + \sqrt{5})}$.

**Challenge 2**

TSA of solid cone $= \pi r^2 + \pi rl.$
TSA of solid hemisphere $= 2\pi r^2$ (curved) $+\pi r^2$ (flat base) $= 3\pi r^2.$
Equate the two surface areas:
$\pi r^2 + \pi rl = 3\pi r^2$
Subtract $\pi r^2$ from both sides:
$\pi rl = 2\pi r^2$
Divide by $\pi r$:
$l = 2r.$
Using Pythagoras for the cone: $h^2 = l^2 - r^2.$
Substitute $l = 2r$:
$h^2 = (2r)^2 - r^2 = 4r^2 - r^2 = 3r^2.$
Square root both sides: $h = \sqrt{3r^2} \implies \mathbf{h = r\sqrt{3}}.$ (Proved)

**Extreme Challenge**
We are given $A = 3B$.
Curved area $A = \pi rl$. Base area $B = \pi r^2$.
Substitute these into the equation:
$\pi rl = 3(\pi r^2)$
Divide by $\pi r$:
$l = 3r$.
Using Pythagoras, find $h$ in terms of $r$:
$h^2 = l^2 - r^2 = (3r)^2 - r^2 = 9r^2 - r^2 = 8r^2$.
$h = \sqrt{8r^2} = 2\sqrt{2}r$.
Now, substitute this $h$ into the formula for the volume of a cone:
Volume $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}(\pi r^2)(2\sqrt{2}r)$.
Notice that $\pi r^2$ is exactly the base area $B$, so we can write:
$V = \frac{1}{3}B(2\sqrt{2}r) = \frac{2\sqrt{2}}{3}Br$.
We need to remove $r$. From $B = \pi r^2$, we can rearrange to find $r$:
$r^2 = \frac{B}{\pi} \implies r = \sqrt{\frac{B}{\pi}}$.
Substitute this $r$ back into the volume expression:
$V = \frac{2\sqrt{2}}{3}B\sqrt{\frac{B}{\pi}}$.
Bring the $\sqrt{2}$ inside the main square root:
$V = \frac{2}{3}B\sqrt{\frac{2B}{\pi}}$. (Proved)