Prime Factorisation

Practice Worksheet (Problem Solving)

I Do

Write 124 as a product of its prime factors.

(Total for Question is 2 marks)

We Do

Write 280 as a product of its prime factors.

(Total for Question is 2 marks)

You Do 1

Write 156 as a product of its prime factors.

(Total for Question is 2 marks)

You Do 2

Write 504 as a product of its prime factors.
Give your answer in index form.

(Total for Question is 3 marks)

Video walkthrough

You Do 3

Write 792 as a product of its prime factors.
Give your answer in index form.

(Total for Question is 3 marks)

Challenge 1

Find the smallest positive integer k such that $124 \times k$ is a perfect square.
You must show clear algebraic working using prime factors.

$k =$ .....

(Total for Question is 3 marks)

Challenge 2

A number N is written as a product of its prime factors:

$$N = 2^a \times 3^b \times 7$$

where a and b are integers.

Given that $N \times 12 = 2^5 \times 3^4 \times 7$, find the values of a and b.

$a =$ ....., $b =$ .....

(Total for Question is 4 marks)

Extreme Challenge

A number M is given by the prime factorisation:

$$M = 2^3 \times 3^2 \times 5$$

Find the smallest whole number w such that $M \times w$ is a perfect cube.
You must show all your working clearly.

$w =$ .....

(Total for Question is 4 marks)

Answers & Worked Solutions

I Do
Divide 124 by the smallest prime number (2):
$124 \div 2 = 62$
Divide 62 by 2:
$62 \div 2 = 31$
31 is a prime number, so it cannot be divided further.
The prime factors are $2, 2$, and 31.
Product: $\mathbf{2 \times 2 \times 31}$ (or $\mathbf{2^2 \times 31}$).

We Do
Divide 280 by 2: $280 \div 2 = 140$
Divide 140 by 2: $140 \div 2 = 70$
Divide 70 by 2: $70 \div 2 = 35$
35 cannot be divided by 2 or 3. Divide by the next prime (5):
$35 \div 5 = 7$
7 is a prime number.
Product: $\mathbf{2 \times 2 \times 2 \times 5 \times 7}$ (or $\mathbf{2^3 \times 5 \times 7}$).

You Do 1
Divide 156 by 2: $156 \div 2 = 78$
Divide 78 by 2: $78 \div 2 = 39$
Divide 39 by 3: $39 \div 3 = 13$
13 is a prime number.
Product: $\mathbf{2 \times 2 \times 3 \times 13}$ (or $\mathbf{2^2 \times 3 \times 13}$).

You Do 2
Divide 504 by 2: 252
Divide 252 by 2: 126
Divide 126 by 2: 63
Divide 63 by 3: 21
Divide 21 by 3: 7 (prime)
The prime factors are $2, 2, 2, 3, 3, 7$.
In index form: $\mathbf{2^3 \times 3^2 \times 7}$.

You Do 3
Divide 792 by 2: 396
Divide 396 by 2: 198
Divide 198 by 2: 99
Divide 99 by 3: 33
Divide 33 by 3: 11 (prime)
The prime factors are $2, 2, 2, 3, 3, 11$.
In index form: $\mathbf{2^3 \times 3^2 \times 11}$.

Challenge 1
From the "I Do" section, we know $124 = 2^2 \times 31$.
For a number to be a perfect square, every prime factor in its prime factorisation must have an **even** index (power).
Currently, the power of 2 is even (2^2), but the power of 31 is odd (31^1).
To make the index of 31 even, we must multiply by at least one more 31 to get 31^2.
Therefore, $k = \mathbf{31}$.

Challenge 2
First, write 12 as a product of its prime factors:
$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$.
Now substitute this into the equation $N \times 12$:
$(2^a \times 3^b \times 7) \times (2^2 \times 3^1) = 2^5 \times 3^4 \times 7$
Using index laws (add the powers for matching bases):
$2^{a+2} \times 3^{b+1} \times 7 = 2^5 \times 3^4 \times 7$
Compare the powers of 2:
$a + 2 = 5 \implies \mathbf{a = 3}$
Compare the powers of 3:
$b + 1 = 4 \implies \mathbf{b = 3}$

Extreme Challenge
$M = 2^3 \times 3^2 \times 5^1$.
For a number to be a perfect cube, every prime factor in its prime factorisation must have an index that is a **multiple of 3** (e.g., 3, 6, 9).
Look at the powers in M:

- Base 2: The power is 3. This is already a multiple of 3, so we do not need to multiply by any 2s.
- Base 3: The power is 2. The next multiple of 3 is 3. We need one more 3 (multiply by 3^1).
- Base 5: The power is 1. The next multiple of 3 is 3. We need two more 5s (multiply by 5^2).

Therefore, the multiplier w must be $3^1 \times 5^2$.
$w = 3 \times 25 = \mathbf{75}$.