Proportion (Powers and Roots)

Practice Worksheet (Problem Solving)

I Do

y is directly proportional to the cube of x.
When $x = 2$, $y = 40$.
Find x when $y = 1080$.

(Total for Question is 3 marks)

We Do

y is directly proportional to the cube of x.
When $x = 3$, $y = 108$.
Find x when $y = 500$.

(Total for Question is 3 marks)

You Do 1

y is directly proportional to the square of x.
When $x = 4$, $y = 48$.
Find the positive value of x when $y = 300$.

(Total for Question is 3 marks)

You Do 2

y is directly proportional to the square root of x.
When $x = 25$, $y = 30$.
Find x when $y = 54$.

Video walkthrough

(Total for Question is 3 marks)

You Do 3

y is inversely proportional to the square of x.
When $x = 2$, $y = 45$.
Find the positive value of x when $y = 5$.

(Total for Question is 3 marks)

Challenge 1

p is directly proportional to q^2.
q is directly proportional to r^3.

When $r = 2$, $p = 288$.
Find the exact value of p when $r = 1$.

(Total for Question is 4 marks)

Challenge 2

y is directly proportional to the square of x.
If the value of x is increased by 20%, work out the percentage increase in the value of y.

.....%

(Total for Question is 4 marks)

Extreme Challenge

y is directly proportional to the square of x.
x is inversely proportional to the cube root of z.

When $x = 2$, $y = 12$.
When $z = 8$, $x = 3$.

Find the exact value of y when $z = 125$.
Show clear algebraic working.

(Total for Question is 5 marks)

Answers & Worked Solutions

I Do
Set up the proportionality equation: $y = kx^3$.
Substitute the given values to find k:
$40 = k \times (2)^3$
$40 = 8k$
$k = \frac{40}{8} = 5$.
The formula connecting y and x is $y = 5x^3$.
To find x when $y = 1080$, substitute y into the formula:
$1080 = 5x^3$
$x^3 = \frac{1080}{5} = 216$
$x = \sqrt[3]{216} = \mathbf{6}$.

We Do
Set up the proportionality equation: $y = kx^3$.
Substitute the given values:
$108 = k \times (3)^3$
$108 = 27k$
$k = \frac{108}{27} = 4$.
The formula is $y = 4x^3$.
Find x when $y = 500$:
$500 = 4x^3$
$x^3 = \frac{500}{4} = 125$
$x = \sqrt[3]{125} = \mathbf{5}$.

You Do 1
Set up the proportionality equation: $y = kx^2$.
$48 = k \times (4)^2 \implies 48 = 16k \implies k = 3$.
The formula is $y = 3x^2$.
Find x when $y = 300$:
$300 = 3x^2 \implies x^2 = 100$.
Since we need the positive value, $x = \sqrt{100} = \mathbf{10}$.

You Do 2
Set up the proportionality equation: $y = k\sqrt{x}$.
$30 = k \times \sqrt{25} \implies 30 = 5k \implies k = 6$.
The formula is $y = 6\sqrt{x}$.
Find x when $y = 54$:
$54 = 6\sqrt{x}$
$\sqrt{x} = \frac{54}{6} = 9$
$x = 9^2 = \mathbf{81}$.

You Do 3

Set up the inverse proportionality equation: $y = \frac{k}{x^2}$.
$45 = \frac{k}{2^2} \implies 45 = \frac{k}{4} \implies k = 45 \times 4 = 180$.
The formula is $y = \frac{180}{x^2}$.
Find x when $y = 5$:
$5 = \frac{180}{x^2} \implies 5x^2 = 180 \implies x^2 = \frac{180}{5} = 36$.
Since we need the positive value, $x = \sqrt{36} = \mathbf{6}$.

Challenge 1

From the statements, $p = aq^2$ and $q = br^3$ (where a and b are constants).
Substitute the expression for q into the equation for p:
$p = a(br^3)^2 = ab^2r^6$.
Since a and b are constants, ab^2 is also a constant, let's call it K.
So, $p = Kr^6$.
Use the given values to find K:
$288 = K \times (2)^6 \implies 288 = 64K \implies K = \frac{288}{64} = 4.5$.
The combined formula is $p = 4.5r^6$.
When $r = 1$: $p = 4.5 \times (1)^6 = \mathbf{4.5}$.

Challenge 2

The relationship is $y = kx^2$.
An increase of 20% means the new value of x is $1.2x$ (or 1.2 times the original).
Substitute this new x into the equation to find the new y:
New $y = k(1.2x)^2$
New $y = k(1.44x^2) = 1.44kx^2$
Since the original y was kx^2, the new y is 1.44 times the original y.
A multiplier of 1.44 represents a **44%** increase.

Extreme Challenge

First, establish the formula for y in terms of x:
$y = kx^2$. Using $x = 2$ and $y = 12$:
$12 = k(2)^2 \implies 12 = 4k \implies k = 3$.
Formula 1: $\mathbf{y = 3x^2}$

Second, establish the formula for x in terms of z:
$x = \frac{m}{\sqrt[3]{z}}$. Using $z = 8$ and $x = 3$:
$3 = \frac{m}{\sqrt[3]{8}} \implies 3 = \frac{m}{2} \implies m = 6$.
Formula 2: $\mathbf{x = \frac{6}{\sqrt[3]{z}}}$

Now, calculate y when $z = 125$:
Step 1: Find x using Formula 2.
$x = \frac{6}{\sqrt[3]{125}} = \frac{6}{5} = 1.2$.
Step 2: Find y using Formula 1.
$y = 3(1.2)^2 = 3(1.44) = \mathbf{4.32}$.