Quadratic Sequences

Practice Worksheet (Problem Solving)

I Do

The nth term of a sequence is $u_n = n^2 - 3n + 5$.

(a) Find the first three terms of the sequence.

(b) Is 113 a term of this sequence? Show algebraic working to justify your answer.

(Total for Question is 4 marks)

We Do

The nth term of a sequence is $u_n = n^2 + 4n - 2$.

(a) Find the first three terms of the sequence.

(b) Is 94 a term of this sequence? Show algebraic working to justify your answer.

(Total for Question is 4 marks)

Video walkthrough

You Do 1

The nth term of a sequence is $u_n = n^2 - 5n + 10$.

(a) Find the first three terms of the sequence.

(b) Is 46 a term of this sequence? Show algebraic working to justify your answer.

(Total for Question is 4 marks)

You Do 2

The nth term of a sequence is $u_n = 2n^2 + n - 5$.

(a) Find the first three terms of the sequence.

(b) Is 131 a term of this sequence? Show algebraic working to justify your answer.

(Total for Question is 4 marks)

You Do 3

The nth term of a sequence is $u_n = n^2 + 6n + 1$.

(a) Find the first three terms of the sequence.

(b) Is 150 a term of this sequence? Show algebraic working to justify your answer.

(Total for Question is 4 marks)

Challenge 1

The nth term of a sequence is given by the expression $3n^2 - 2n + k$, where k is an integer.
The 4th term of the sequence is 50.

(a) Find the value of k.

$k =$ .....

(b) Is 186 a term of this sequence? Show algebraic working to justify your answer.

(Total for Question is 6 marks)

Challenge 2

The nth term of a sequence is $u_n = n^2 - 8n + 20$.

By completing the square, or otherwise, find the smallest term in the sequence and the position n at which it occurs.
Show clear algebraic working.

Smallest term = ... at $n =$ .....

(Total for Question is 4 marks)

Extreme Challenge

Sequence A has an nth term given by $u_n = n^2 + 5n - 8$.
Sequence B has an nth term given by $v_n = 2n^2 - 3n + 7$.

For certain positions in the sequence, the term in Sequence A is exactly equal to the term in Sequence B.

Find the values of these terms and the positions n at which they occur.
You must show all your algebraic working clearly.

(Total for Question is 6 marks)

Answers & Worked Solutions

I Do

(a) Substitute $n = 1, n = 2$, and $n = 3$ into $n^2 - 3n + 5$:

$n = 1 : (1)^2 - 3(1) + 5 = 1 - 3 + 5 = 3$

$n = 2 : (2)^2 - 3(2) + 5 = 4 - 6 + 5 = 3$

$n = 3 : (3)^2 - 3(3) + 5 = 9 - 9 + 5 = 5$

First three terms are: $\mathbf{3}, \mathbf{3}, \mathbf{5}$.

(b) Set the expression equal to 113 and solve for n:

$n^2 - 3n + 5 = 113$

$n^2 - 3n - 108 = 0$

Factorise the quadratic: $(n - 12)(n + 9) = 0$

Since n represents a position in a sequence, it must be a positive integer.

$n = 12$.

Yes, 113 is the 12th term.

We Do

(a) Substitute $n = 1, 2, 3$ into $n^2 + 4n - 2$:

$n = 1 : 1 + 4 - 2 = 3$

$n = 2 : 4 + 8 - 2 = 10$

$n = 3 : 9 + 12 - 2 = 19$

First three terms are: $\mathbf{3}, \mathbf{10}, \mathbf{19}$.

(b) Set equal to 94:

$n^2 + 4n - 2 = 94$

$n^2 + 4n - 96 = 0$

$(n + 12)(n - 8) = 0$

$n = 8$ (reject $n = -12$).

Yes, 94 is the 8th term.

You Do 1

(a) Substitute $n = 1, 2, 3$ into $n^2 - 5n + 10$:

$n = 1 : 1 - 5 + 10 = 6$

$n = 2 : 4 - 10 + 10 = 4$

$n = 3 : 9 - 15 + 10 = 4$

First three terms are: $\mathbf{6}, \mathbf{4}, \mathbf{4}$.

(b) Set equal to 46:

$n^2 - 5n + 10 = 46$

$n^2 - 5n - 36 = 0$

$(n - 9)(n + 4) = 0$

$n = 9$ (reject $n = -4$).

Yes, 46 is the 9th term.

You Do 2
(a) Substitute $n = 1, 2, 3$ into $2n^2 + n - 5$:
$n = 1 : 2(1)^2 + 1 - 5 = 2 + 1 - 5 = -2$
$n = 2 : 2(2)^2 + 2 - 5 = 8 + 2 - 5 = 5$
$n = 3 : 2(3)^2 + 3 - 5 = 18 + 3 - 5 = 16$
First three terms are: $\mathbf{-2}, \mathbf{5}, \mathbf{16}$.
(b) Set equal to 131:
$2n^2 + n - 5 = 131$
$2n^2 + n - 136 = 0$
Factorise: $(2n + 17)(n - 8) = 0$
$n = 8$ (reject $n = -8.5$).
Yes, 131 is the 8th term.

You Do 3
(a) Substitute $n = 1, 2, 3$ into $n^2 + 6n + 1$:
$n = 1 : 1 + 6 + 1 = 8$
$n = 2 : 4 + 12 + 1 = 17$
$n = 3 : 9 + 18 + 1 = 28$
First three terms are: $\mathbf{8}, \mathbf{17}, \mathbf{28}$.
(b) Set equal to 150:
$n^2 + 6n + 1 = 150$
$n^2 + 6n - 149 = 0$
Using the quadratic formula: $n = \frac{-6 \pm \sqrt{36 - 4(1)(-149)}}{2} = \frac{-6 \pm \sqrt{632}}{2}$
Since 632 is not a perfect square (approx 25.14), n is not an integer.
No, 150 is not a term in the sequence.

Challenge 1
(a) We are told that when $n = 4$, the term is 50.
Substitute $n = 4$ into $3n^2 - 2n + k = 50$:
$3(16) - 2(4) + k = 50$
$48 - 8 + k = 50$
$40 + k = 50 \implies \mathbf{k = 10}$.
(b) The nth term is $u_n = 3n^2 - 2n + 10$.
Set equal to 186:
$3n^2 - 2n + 10 = 186$
$3n^2 - 2n - 176 = 0$
Factorise: $(3n + 22)(n - 8) = 0$
$n = 8$ (reject $n = -22/3$).
Yes, 186 is the 8th term.

Challenge 2
To find the minimum value of the quadratic $n^2 - 8n + 20$, we can complete the square:
$n^2 - 8n + 20 = (n - 4)^2 - 16 + 20 = (n - 4)^2 + 4$
The minimum value occurs when the squared bracket is zero.
This happens when $n - 4 = 0 \implies n = 4$.
When $n = 4$, the value of the term is $(4 - 4)^2 + 4 = 4$.
Smallest term is **4**, occurring at $\mathbf{n = 4}$.
(Alternatively, calculate the first few terms: $13, 8, 5, 4, 5 \ldots$ and spot the minimum).

Extreme Challenge

Set the two nth term expressions equal to each other to find the position(s) n where they match:

$u_n = v_n$

$n^2 + 5n - 8 = 2n^2 - 3n + 7$

Rearrange to form a quadratic equal to zero (move everything to the right side):

$0 = 2n^2 - n^2 - 3n - 5n + 7 + 8$

$0 = n^2 - 8n + 15$

Factorise the quadratic:

$0 = (n - 3)(n - 5)$

This gives two valid positions: $\mathbf{n = 3}$ and $\mathbf{n = 5}$.

Now substitute these back into either sequence to find the values of the terms:

For $n = 3$: $u_3 = (3)^2 + 5(3) - 8 = 9 + 15 - 8 = \mathbf{16}$.

For $n = 5$: $u_5 = (5)^2 + 5(5) - 8 = 25 + 25 - 8 = \mathbf{42}$.

The terms are 16 (at position n=3) and 42 (at position n=5).