Simplifying Algebraic Fractions

Practice Worksheet (Problem Solving)

I Do

Simplify fully.

$$\frac{2x^2 + 5x - 12}{2x^2 + 7x - 4}$$

(Total for Question is 3 marks)

We Do

Simplify fully.

$$\frac{3x^2 + 5x - 2}{3x^2 - 7x + 2}$$

(Total for Question is 3 marks)

You Do 1

Simplify fully.

$$\frac{2x^2 + 7x + 3}{2x^2 + 5x + 2}$$

(Total for Question is 3 marks)

Video walkthrough

You Do 2

Simplify fully.

$$\frac{5x^2 - 13x - 6}{5x^2 - 8x - 4}$$

(Total for Question is 3 marks)

You Do 3

Simplify fully.

$$\frac{4x^2 - 9}{2x^2 + 7x + 6}$$

(Total for Question is 3 marks)

Challenge 1

Show that

$$\frac{x^2 - 16}{2x^2 - 5x - 12} \times \frac{4x^2 - 9}{2x^2 - 11x + 12}$$

simplifies to the form $\frac{ax+b}{cx+d}$ where a, b, c, and d are integers.

(Total for Question is 4 marks)

Challenge 2

Express as a single fraction in its simplest form.

$$\frac{3}{x^2 + 5x + 4} - \frac{2}{x^2 + 3x + 2}$$

(Total for Question is 4 marks)

Extreme Challenge

Solve the equation

$$\frac{3}{x-1} - \frac{2}{x+1} = 1$$

Show clear algebraic working.

$x =$ .....

(Total for Question is 5 marks)

Answers & Worked Solutions

I Do

Factorise the numerator: $2x^2 + 5x - 12$
Multiply $2 \times -12 = -24$. We need two numbers that multiply to -24 and add to 5. These are 8 and -3.
$2x^2 + 8x - 3x - 12 = 2x(x + 4) - 3(x + 4) = (2x - 3)(x + 4)$.
Factorise the denominator: $2x^2 + 7x - 4$
Multiply $2 \times -4 = -8$. Numbers are 8 and -1.
$2x^2 + 8x - x - 4 = 2x(x + 4) - 1(x + 4) = (2x - 1)(x + 4)$.
Substitute back into the fraction:
$\frac{(2x-3)(x+4)}{(2x-1)(x+4)}$
Cancel the common factor $(x + 4)$:
$\mathbf{\frac{2x-3}{2x-1}}$

We Do

Factorise the numerator: $3x^2 + 5x - 2$
Numbers that multiply to -6 and add to 5 are 6 and -1.
$3x^2 + 6x - x - 2 = 3x(x + 2) - 1(x + 2) = (3x - 1)(x + 2)$.
Factorise the denominator: $3x^2 - 7x + 2$
Numbers that multiply to 6 and add to -7 are -6 and -1.
$3x^2 - 6x - x + 2 = 3x(x - 2) - 1(x - 2) = (3x - 1)(x - 2)$.
Substitute back:
$\frac{(3x-1)(x+2)}{(3x-1)(x-2)}$
Cancel $(3x - 1)$:
$\mathbf{\frac{x+2}{x-2}}$

You Do 1

Numerator: $2x^2 + 7x + 3 = (2x + 1)(x + 3)$
Denominator: $2x^2 + 5x + 2 = (2x + 1)(x + 2)$
Substitute back:
$\frac{(2x+1)(x+3)}{(2x+1)(x+2)}$
Cancel $(2x + 1)$:
$\mathbf{\frac{x+3}{x+2}}$

You Do 2

Numerator: $5x^2 - 13x - 6$
Numbers multiplying to -30 and adding to -13 are -15 and 2.
$5x^2 - 15x + 2x - 6 = 5x(x - 3) + 2(x - 3) = (5x + 2)(x - 3)$.
Denominator: $5x^2 - 8x - 4$
Numbers multiplying to -20 and adding to -8 are -10 and 2.
$5x^2 - 10x + 2x - 4 = 5x(x - 2) + 2(x - 2) = (5x + 2)(x - 2)$.
Substitute back and cancel $(5x + 2)$:
$\mathbf{\frac{x-3}{x-2}}$

You Do 3

Numerator: $4x^2 - 9$ is a Difference of Two Squares.
$(2x)^2 - (3)^2 = (2x - 3)(2x + 3)$.
Denominator: $2x^2 + 7x + 6$
Numbers multiplying to 12 and adding to 7 are 4 and 3.
$2x^2 + 4x + 3x + 6 = 2x(x + 2) + 3(x + 2) = (2x + 3)(x + 2)$.
Substitute back:
$\frac{(2x-3)(2x+3)}{(2x+3)(x+2)}$
Cancel $(2x + 3)$:
$\mathbf{\frac{2x-3}{x+2}}$

Challenge 1

Factorise all four quadratics first.
Fraction 1 Num: $x^2 - 16 = (x - 4)(x + 4)$
Fraction 1 Den: $2x^2 - 5x - 12 = (2x + 3)(x - 4)$
Fraction 2 Num: $4x^2 - 9 = (2x - 3)(2x + 3)$
Fraction 2 Den: $2x^2 - 11x + 12 = (2x - 3)(x - 4)$
Write the full multiplication:
$\frac{(x-4)(x+4)}{(2x+3)(x-4)} \times \frac{(2x-3)(2x+3)}{(2x-3)(x-4)}$
Cancel $(x - 4)$ in the first fraction.
Cancel $(2x - 3)$ in the second fraction.
We are left with:
$\frac{x+4}{2x+3} \times \frac{2x+3}{x-4}$
Now cross-cancel the $(2x + 3)$ terms.
Result: $\mathbf{\frac{x+4}{x-4}}$

Challenge 2

Factorise the denominators first:
$x^2 + 5x + 4 = (x + 1)(x + 4)$
$x^2 + 3x + 2 = (x + 1)(x + 2)$
The expression becomes:
$\frac{3}{(x+1)(x+4)} - \frac{2}{(x+1)(x+2)}$
Find the lowest common denominator (LCD), which is $(x + 1)(x + 2)(x + 4)$.
Multiply the first fraction by $\frac{x+2}{x+2}$:
$\frac{3(x+2)}{(x+1)(x+2)(x+4)}$
Multiply the second fraction by $\frac{x+4}{x+4}$:
$\frac{2(x+4)}{(x+1)(x+2)(x+4)}$
Subtract the numerators:
$3(x + 2) - 2(x + 4) = 3x + 6 - 2x - 8 = x - 2$.
Final single fraction:
$\mathbf{\frac{x-2}{(x+1)(x+2)(x+4)}}$

Extreme Challenge

$\frac{3}{x-1} - \frac{2}{x+1} = 1$

First, combine the fractions on the left-hand side.

The common denominator is $(x-1)(x+1)$.

$\frac{3(x+1)}{(x-1)(x+1)} - \frac{2(x-1)}{(x-1)(x+1)} = 1$

Simplify the numerator:

$3x + 3 - (2x - 2) = 3x + 3 - 2x + 2 = x + 5.$

The equation becomes:

$\frac{x+5}{(x-1)(x+1)} = 1$

Multiply both sides by $(x-1)(x+1)$:

$x + 5 = (x-1)(x+1)$

Expand the right-hand side (Difference of Two Squares):

$x + 5 = x^2 - 1$

Rearrange to form a quadratic equal to zero:

$x^2 - x - 6 = 0$

Factorise the quadratic:

$(x-3)(x+2) = 0$

The solutions are:

$\mathbf{x = 3}$ or $\mathbf{x = -2}$