Equation of a Straight Line

Practice Worksheet (Problem Solving)

I Do

Find the equation of the line that passes through $(2, 5)$ and $(6, 17)$.
Give your answer in the form $y = mx + c$.

(Total for Question is 3 marks)

We Do

Find the equation of the line that passes through $(1, 8)$ and $(4, 2)$.
Give your answer in the form $y = mx + c$.

(Total for Question is 3 marks)

You Do 1

Find the equation of the line that passes through $(3, 7)$ and $(5, 15)$.
Give your answer in the form $y = mx + c$.

(Total for Question is 3 marks)

You Do 2

Find the equation of the line that passes through $(-2, 10)$ and $(2, -2)$.
Give your answer in the form $y = mx + c$.

(Total for Question is 3 marks)

Video walkthrough

You Do 3

Find the equation of the line that passes through $(2, 1)$ and $(6, 3)$.
Give your answer in the form $y = mx + c$.

(Total for Question is 3 marks)

Challenge 1

Line L_1 has equation $y = 2x + 5$.
Line L_2 is parallel to L_1 and passes through the point $(4, -1)$.

Find the equation of L_2.
Give your answer in the form $y = mx + c$.

(Total for Question is 3 marks)

Challenge 2

Line L_3 has equation $y = 3x - 2$.
Line L_4 is perpendicular to L_3 and passes through the point $(6, 5)$.

Find the equation of L_4.
Give your answer in the form $y = mx + c$.

(Total for Question is 4 marks)

Extreme Challenge

Find the equation of the perpendicular bisector of the line segment joining the points $A(-4, 2)$ and $B(2, 10)$.

Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers.
Show clear algebraic working.

(Total for Question is 5 marks)

Answers & Worked Solutions

I Do

First, find the gradient (m):
$m = \frac{\text{change in } y}{\text{change in } x} = \frac{17-5}{6-2} = \frac{12}{4} = 3$
The equation of the line is $y = 3x + c$.
Substitute one of the points, e.g., $(2, 5)$, to find c:
$5 = 3(2) + c$
$5 = 6 + c$
$c = -1$
Equation: $\mathbf{y = 3x - 1}$

We Do

Find the gradient (m):
$m = \frac{2-8}{4-1} = \frac{-6}{3} = -2$
The equation of the line is $y = -2x + c$.
Substitute the point $(1, 8)$:
$8 = -2(1) + c$
$8 = -2 + c$
$c = 10$
Equation: $\mathbf{y = -2x + 10}$

You Do 1

Gradient $m = \frac{15-7}{5-3} = \frac{8}{2} = 4$
Equation is $y = 4x + c$.
Substitute $(3, 7)$:
$7 = 4(3) + c \implies 7 = 12 + c \implies c = -5$
Equation: $\mathbf{y = 4x - 5}$

You Do 2

Gradient $m = \frac{-2-10}{2-(-2)} = \frac{-12}{4} = -3$
Equation is $y = -3x + c$.
Substitute $(2, -2)$:
$-2 = -3(2) + c \implies -2 = -6 + c \implies c = 4$
Equation: $\mathbf{y = -3x + 4}$

You Do 3

Gradient $m = \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}$
Equation is $y = \frac{1}{2}x + c$.
Substitute $(2, 1)$:
$1 = \frac{1}{2}(2) + c \implies 1 = 1 + c \implies c = 0$
Equation: $\mathbf{y = \frac{1}{2}x}$ (or $y = 0.5x$)

Challenge 1

Parallel lines have the same gradient.
The gradient of L_1 is 2, so the gradient of L_2 is also 2.
Equation of L_2 is $y = 2x + c$.
Substitute the given point $(4, -1)$:
$-1 = 2(4) + c$
$-1 = 8 + c \implies c = -9$
Equation: $\mathbf{y = 2x - 9}$

Challenge 2

The gradient of L_3 is 3.
Perpendicular lines have gradients that multiply to make -1 (the negative reciprocal).
Gradient of L_4 is $-\frac{1}{3}$.
Equation of L_4 is $y = -\frac{1}{3}x + c$.
Substitute the given point $(6, 5)$:
$5 = -\frac{1}{3}(6) + c$
$5 = -2 + c \implies c = 7$
Equation: $\mathbf{y = -\frac{1}{3}x + 7}$

Extreme Challenge

1. Find the midpoint of A and B:
$M = \left(\frac{-4+2}{2}, \frac{2+10}{2}\right) = \left(\frac{-2}{2}, \frac{12}{2}\right) = (-1, 6)$.
2. Find the gradient of line segment AB:
$m_{AB} = \frac{10-2}{2-(-4)} = \frac{8}{6} = \frac{4}{3}$.
3. Find the perpendicular gradient:
$m_{\perp} = -\frac{3}{4}$.
4. Construct the equation of the line using the midpoint $(-1, 6)$:
$y = -\frac{3}{4}x + c$
$6 = -\frac{3}{4}(-1) + c \implies 6 = \frac{3}{4} + c \implies c = \frac{24}{4} - \frac{3}{4} = \frac{21}{4}$.
$y = -\frac{3}{4}x + \frac{21}{4}$
Multiply everything by 4 to remove fractions:
$4y = -3x + 21$
Rearrange into the form $ax + by + c = 0$:

$\mathbf{3x + 4y - 21 = 0}$