Vector Geometry

Practice Worksheet (Problem Solving)

I Do

OABC is a parallelogram. Vector $\overrightarrow{OA} = \mathbf{a}$ and vector $\overrightarrow{OC} = \mathbf{c}$.
X lies on AC such that AX : XC = 1 : 3.
Find vector $\overrightarrow{OX}$ in terms of **a** and **c**. Give your answer in its simplest form.

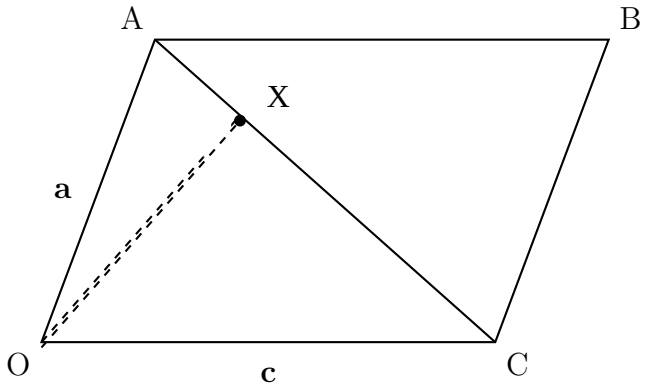


Diagram not to scale

(Total for Question is 3 marks)

Video walkthrough

We Do

OPQR is a parallelogram. Vector $\overrightarrow{OP} = \mathbf{p}$ and vector $\overrightarrow{OR} = \mathbf{r}$.
Y lies on PR such that PY : YR = 2 : 3.
Find vector $\overrightarrow{OY}$ in terms of $\mathbf{p}$ and $\mathbf{r}$. Give your answer in its simplest form.

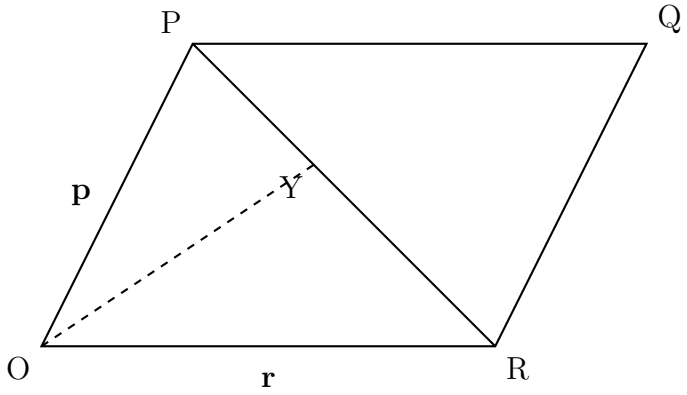


Diagram not to scale

(Total for Question is 3 marks)

You Do 1

ODEF is a parallelogram. Vector $\overrightarrow{OD} = \mathbf{d}$ and vector $\overrightarrow{OF} = \mathbf{f}$.
Z lies on DF such that DZ : ZF = 1 : 4.
Find vector $\overrightarrow{OZ}$ in terms of **d** and **f**. Give your answer in its simplest form.

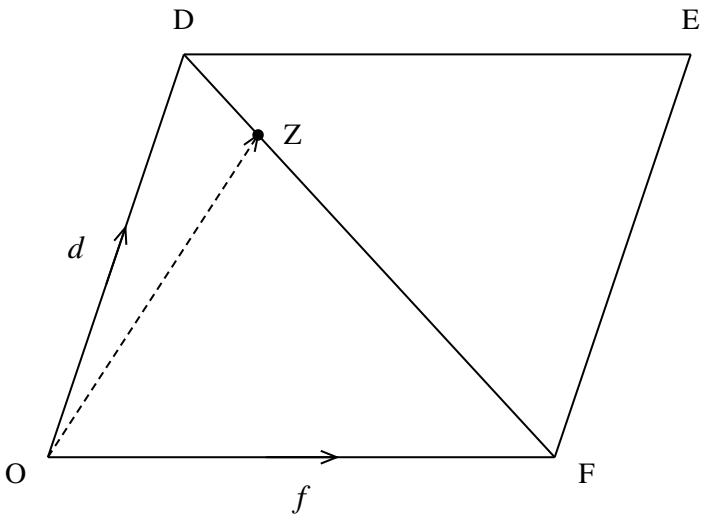


Diagram not to scale

(Total for Question is 3 marks)

You Do 2

OAB is a triangle.
Vector $\overrightarrow{OA} = \mathbf{a}$ and vector $\overrightarrow{OB} = \mathbf{b}$.
M is the midpoint of AB.
Find vector $\overrightarrow{OM}$ in terms of **a** and **b**. Give your answer in its simplest form.

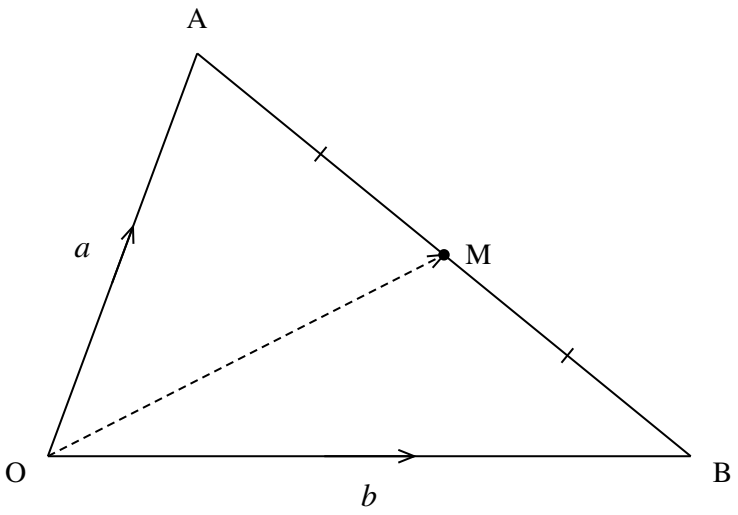


Diagram not to scale

(Total for Question is 3 marks)

You Do 3

OPQ is a triangle.
Vector $\overrightarrow{OP} = 3\mathbf{p}$ and vector $\overrightarrow{OQ} = 3\mathbf{q}$.
X lies on PQ such that PX : XQ = 2 : 1.
Find vector $\overrightarrow{OX}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. Give your answer in its simplest form.

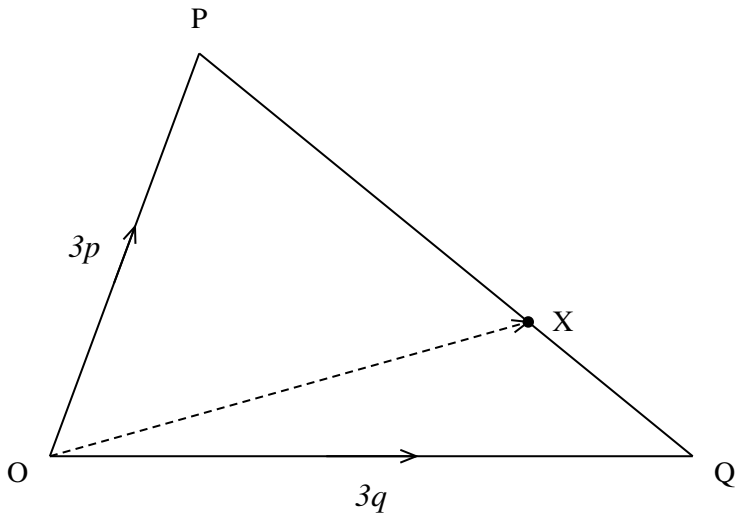


Diagram not to scale

(Total for Question is 3 marks)

Challenge 1

OABC is a parallelogram.
Vector $\overrightarrow{OA} = \mathbf{a}$ and vector $\overrightarrow{OC} = \mathbf{c}$.
M is the midpoint of CB.
N is a point on AB such that AN : NB = 2 : 1.

Find vector $\overrightarrow{MN}$ in terms of **a** and **c**.
Give your answer in its simplest form.

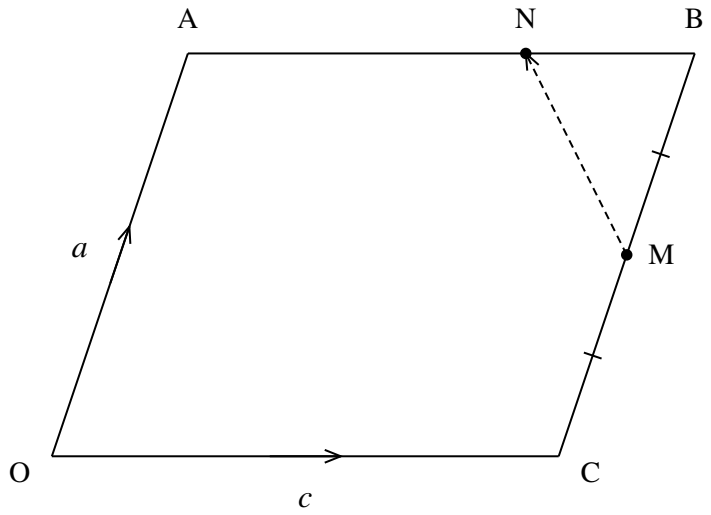


Diagram not to scale

(Total for Question is 4 marks)

Challenge 2

OAB is a triangle.
Vector $\overrightarrow{OA} = \mathbf{a}$ and vector $\overrightarrow{OB} = \mathbf{b}$.
P is a point on OA such that OP : PA = 1 : 2.
Q is a point on OB such that OQ : QB = 1 : 2.

Use vectors to prove that the line segment PQ is parallel to the line segment AB.

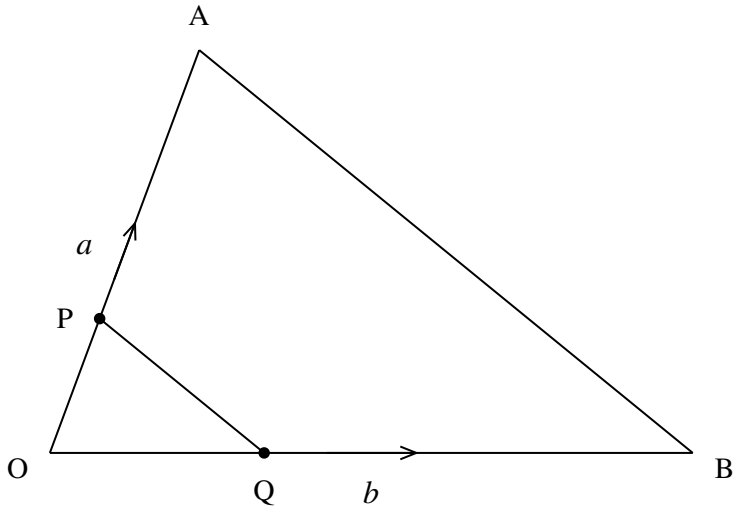


Diagram not to scale

(Total for Question is 4 marks)

Extreme Challenge

OABC is a parallelogram.
Vector $\overrightarrow{OA} = \mathbf{a}$ and vector $\overrightarrow{OC} = \mathbf{c}$.
M is the midpoint of BC.
N is a point on AC such that AN : NC = 2 : 1.

Use vectors to prove that the points O, N, and M lie on the same straight line (they are collinear).
You must show all your working clearly.

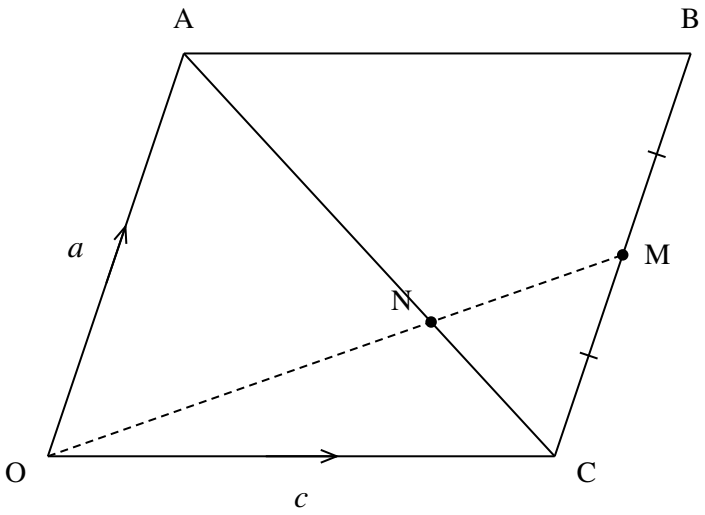


Diagram not to scale

(Total for Question is 6 marks)

Answers & Worked Solutions

I Do
First, find the vector for the full diagonal $\overrightarrow{AC}$:
$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\mathbf{a} + \mathbf{c} = \mathbf{c} - \mathbf{a}$.
Since AX : XC = 1 : 3, the total number of parts is 4.
Therefore, $\overrightarrow{AX}$ is $\frac{1}{4}$ of $\overrightarrow{AC}$:
$\overrightarrow{AX} = \frac{1}{4}(\mathbf{c} - \mathbf{a})$.
Now, find $\overrightarrow{OX}$ by travelling from O to A, then A to X:
$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX} = \mathbf{a} + \frac{1}{4}(\mathbf{c} - \mathbf{a})$.
Expand and simplify:
$\overrightarrow{OX} = \mathbf{a} + \frac{1}{4}\mathbf{c} - \frac{1}{4}\mathbf{a} = \mathbf{\frac{3}{4}a} + \mathbf{\frac{1}{4}c}$.

We Do
$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -\mathbf{p} + \mathbf{r} = \mathbf{r} - \mathbf{p}$.
PY : YR = 2 : 3, so the total parts is 5.
$\overrightarrow{PY} = \frac{2}{5}(\mathbf{r} - \mathbf{p})$.
$\overrightarrow{OY} = \overrightarrow{OP} + \overrightarrow{PY} = \mathbf{p} + \frac{2}{5}(\mathbf{r} - \mathbf{p})$.
Expand and simplify:
$\overrightarrow{OY} = \mathbf{p} + \frac{2}{5}\mathbf{r} - \frac{2}{5}\mathbf{p} = \mathbf{\frac{3}{5}p} + \mathbf{\frac{2}{5}r}$.

You Do 1
$\overrightarrow{DF} = \mathbf{f} - \mathbf{d}$.
DZ : ZF = 1 : 4, so total parts = 5.
$\overrightarrow{DZ} = \frac{1}{5}(\mathbf{f} - \mathbf{d})$.
$\overrightarrow{OZ} = \overrightarrow{OD} + \overrightarrow{DZ} = \mathbf{d} + \frac{1}{5}(\mathbf{f} - \mathbf{d})$.
Expand and simplify:
$\overrightarrow{OZ} = \mathbf{d} + \frac{1}{5}\mathbf{f} - \frac{1}{5}\mathbf{d} = \mathbf{\frac{4}{5}d} + \mathbf{\frac{1}{5}f}$.

You Do 2
$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \mathbf{b} - \mathbf{a}$.
M is the midpoint, so $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$.
$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$.
Expand and simplify:
$\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} = \mathbf{\frac{1}{2}a} + \mathbf{\frac{1}{2}b}$.

You Do 3
$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -3\mathbf{p} + 3\mathbf{q} = 3\mathbf{q} - 3\mathbf{p}$.
PX : XQ = 2 : 1, so total parts = 3.
$\overrightarrow{PX} = \frac{2}{3}(3\mathbf{q} - 3\mathbf{p}) = 2\mathbf{q} - 2\mathbf{p}$.
$\overrightarrow{OX} = \overrightarrow{OP} + \overrightarrow{PX} = 3\mathbf{p} + (2\mathbf{q} - 2\mathbf{p})$.
Simplify:
$\overrightarrow{OX} = \mathbf{p} + \mathbf{2q}$.

Challenge 1

In the parallelogram, OB = a + c.

M is the midpoint of CB, so $OM = c + \frac{1}{2}a$.

AN : NB = 2 : 1, so $ON = a + \frac{2}{3}c$.

$MN = ON - OM = a + \frac{2}{3}c - (c + \frac{1}{2}a)$.

Therefore $MN = \frac{1}{2}a - \frac{1}{3}c$.

Challenge 2

First, find vector $\overrightarrow{AB}$:
$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \mathbf{b} - \mathbf{a}$.
Next, find the vectors for OP and OQ:
$OP : PA = 1 : 2 \implies \overrightarrow{OP} = \frac{1}{3}\mathbf{a}$.
$OQ : QB = 1 : 2 \implies \overrightarrow{OQ} = \frac{1}{3}\mathbf{b}$.
Now, find vector $\overrightarrow{PQ}$:
$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.
Factor out the $\frac{1}{3}$:
$\overrightarrow{PQ} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$.
Since $\overrightarrow{PQ}$ is a scalar multiple ($\frac{1}{3}$) of $\overrightarrow{AB}$, the two vectors are parallel.

Extreme Challenge

AC = c − a.

AN : NC = 2 : 1, so $AN = \frac{2}{3}(c - a)$.

$ON = OA + AN = a + \frac{2}{3}(c - a) = \frac{1}{3}a + \frac{2}{3}c$.

M is the midpoint of BC. Since OC = c and OB = a + c,

$OM = \frac{1}{2}(OC + OB) = \frac{1}{2}(c + a + c) = \frac{1}{2}a + c$.

Hence $OM = \frac{3}{2}ON$.

These vectors have the common origin O, and one is a scalar multiple of the other.

Therefore O, N and M are collinear.