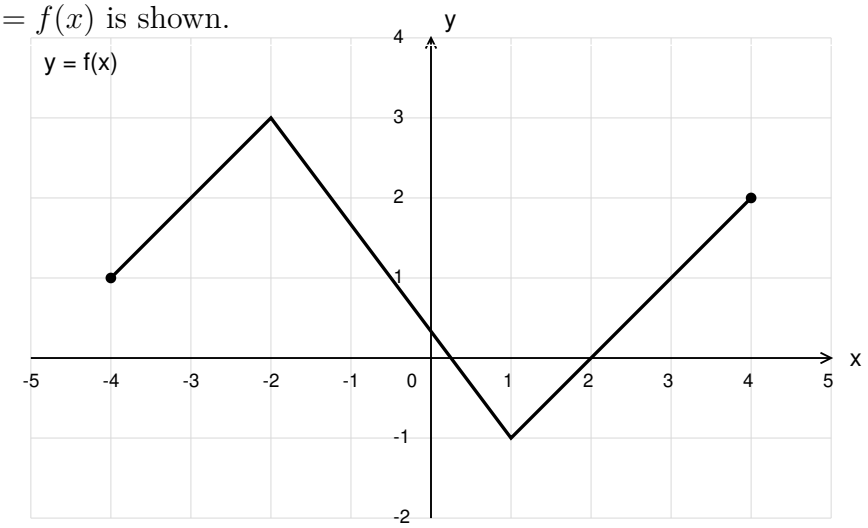
Transformations of Functions

Practice Worksheet (Problem Solving)

I Do

The graph of $y = f(x)$ is shown.



The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ by the vector

$$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

Write $g(x)$ in terms of $f(x)$.

.....

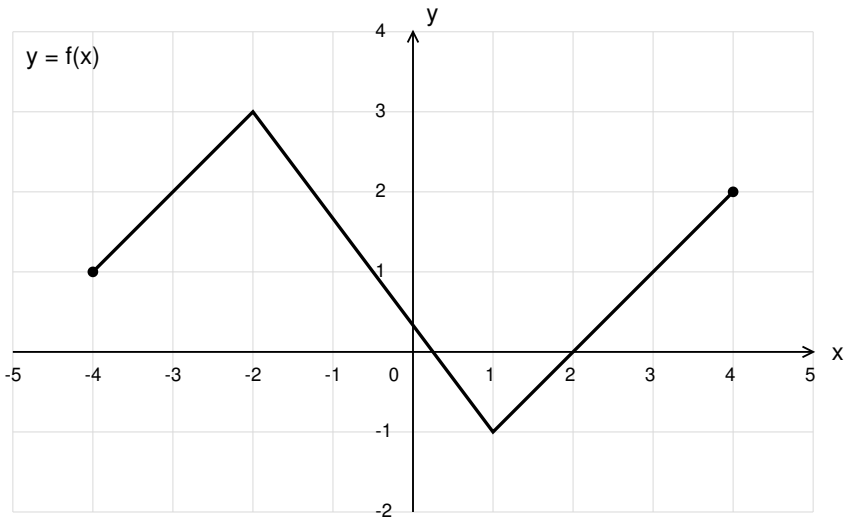
The graph of $y = h(x)$ is obtained from $y = f(x)$ by a stretch parallel to the x-axis with scale factor 2.

Write $h(x)$ in terms of $f(x)$.

.....

(Total for Question is 4 marks)

We Do



The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ by the vector

$$\begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

Write $g(x)$ in terms of $f(x)$.

.....

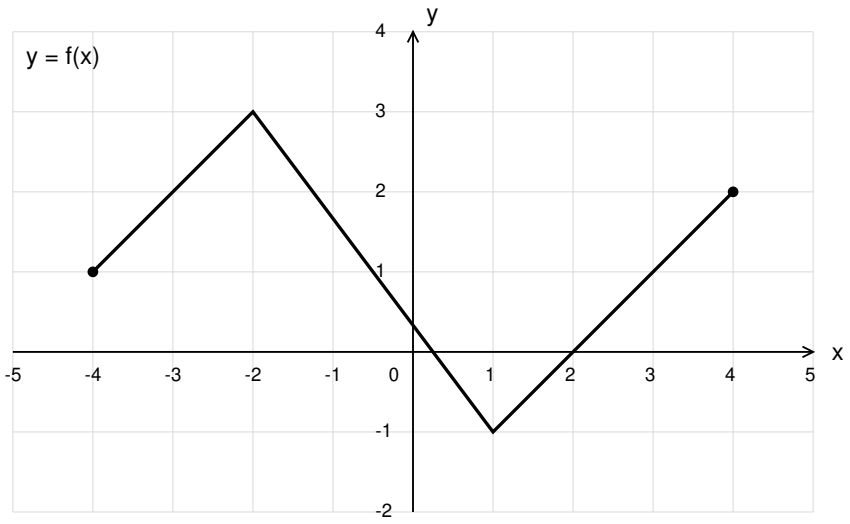
The graph of $y = h(x)$ is obtained from $y = f(x)$ by a stretch parallel to the y-axis with scale factor 3.

Write $h(x)$ in terms of $f(x)$.

.....

(Total for Question is 4 marks)

You Do 1



The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ by the vector

$$\begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

Write $g(x)$ in terms of $f(x)$.

.....

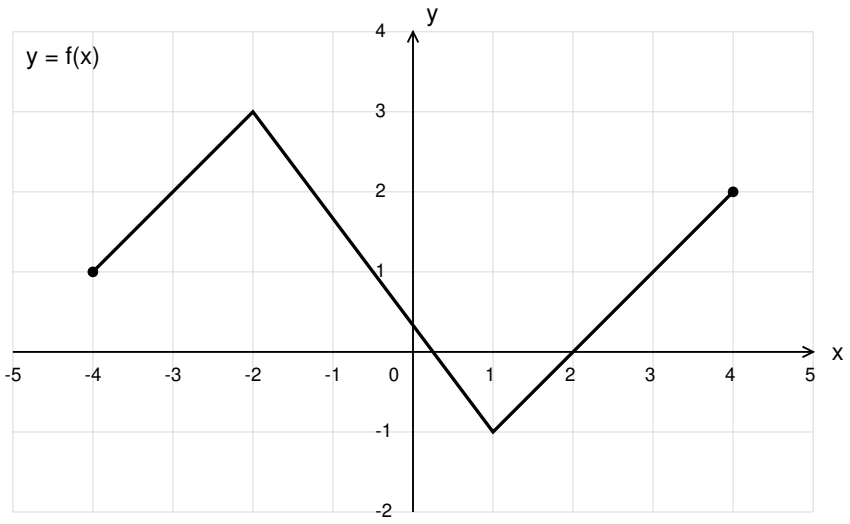
The graph of $y = h(x)$ is obtained from $y = f(x)$ by a stretch parallel to the x-axis with scale factor $\frac{1}{3}$.

Write $h(x)$ in terms of $f(x)$.

.....

(Total for Question is 4 marks)

You Do 2



The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ by the vector

$$\begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

Write $g(x)$ in terms of $f(x)$.

.....

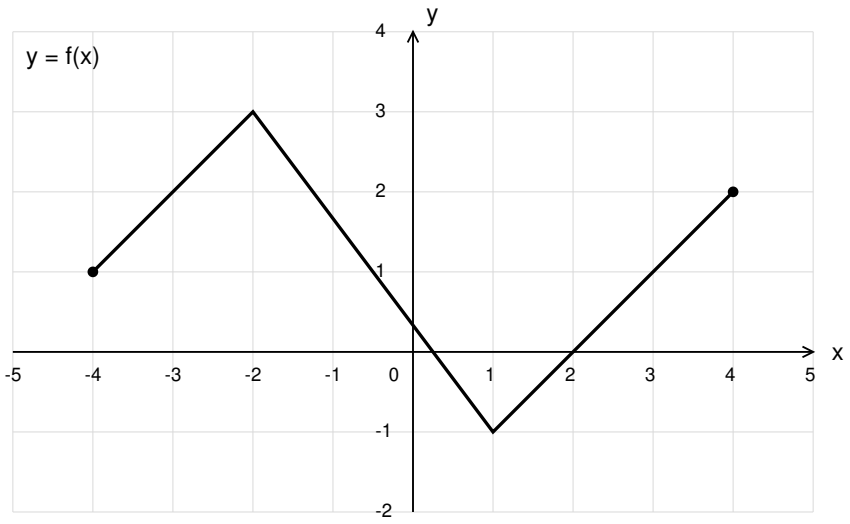
The graph of $y = h(x)$ is obtained from $y = f(x)$ by a reflection in the x-axis.

Write $h(x)$ in terms of $f(x)$.

.....

(Total for Question is 4 marks)

You Do 3



The graph of $y = g(x)$ is obtained from $y = f(x)$ by a reflection in the y-axis.

Write $g(x)$ in terms of $f(x)$.

.....

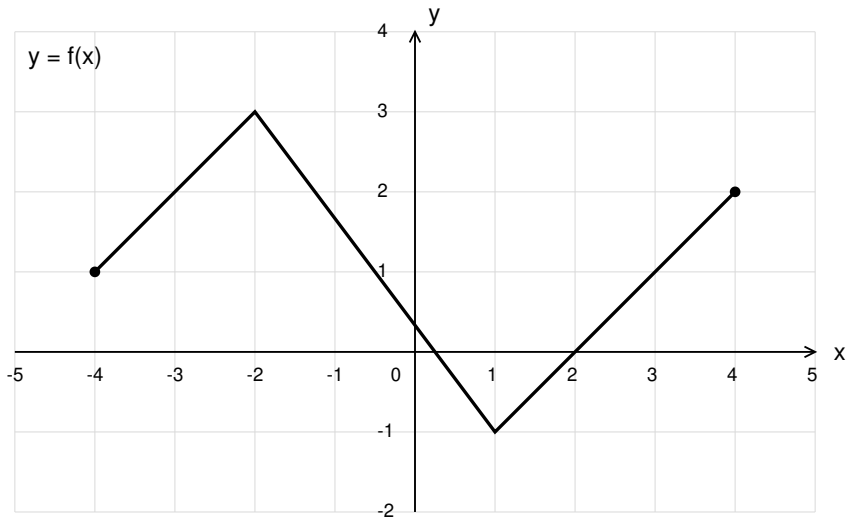
The graph of $y = h(x)$ is obtained from $y = f(x)$ by a stretch parallel to the y-axis with scale factor 0.5.

Write $h(x)$ in terms of $f(x)$.

.....

(Total for Question is 4 marks)

Challenge 1



The graph of $y = g(x)$ is a transformation of the graph of $y = f(x)$.

The equation of $g(x)$ is given by:

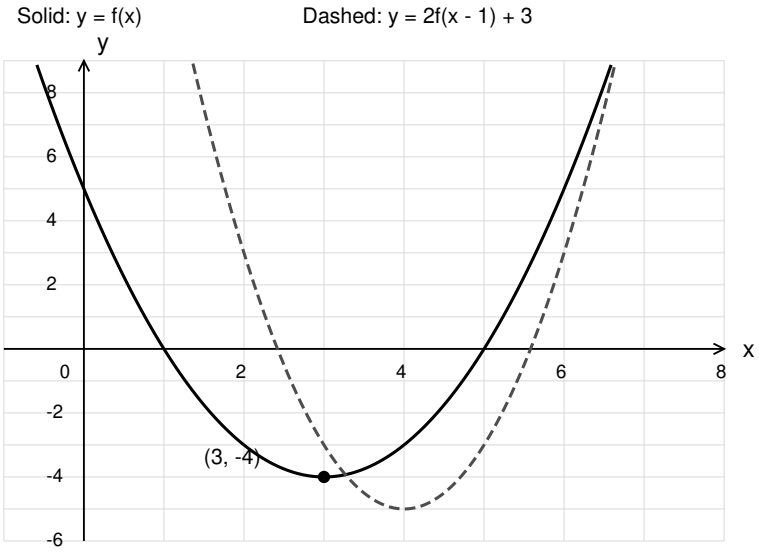
$$y = -f(x + 2) + 5$$

Describe fully the sequence of three transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$.

.....

(Total for Question is 4 marks)

Challenge 2



The graph of $y = f(x)$ has a minimum turning point at the coordinates $(3, -4)$.

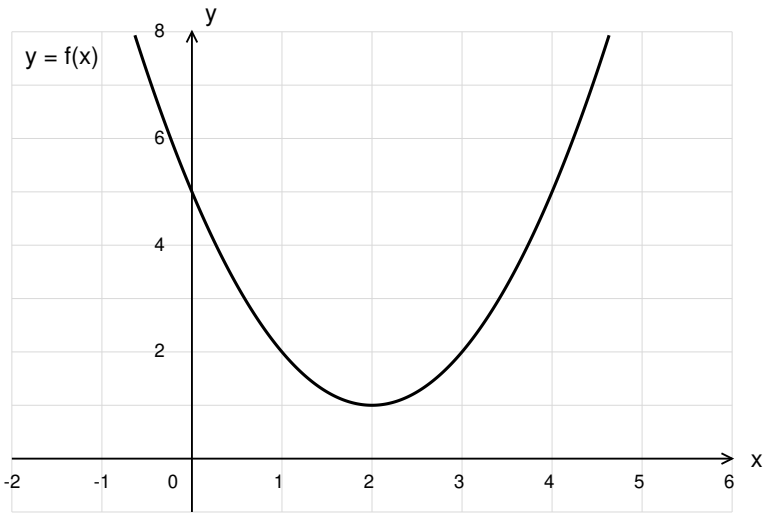
The graph of $y = 2f(x - 1) + 3$ is plotted.

Find the exact coordinates of the minimum turning point of this new graph.
Show clear algebraic reasoning.

.....

(Total for Question is 4 marks)

Extreme Challenge



A quadratic function is defined as $f(x) = x^2 - 4x + 5$.

The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ by the vector

$$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Find an expression for $g(x)$ in the form $ax^2 + bx + c$, where a, b, and c are integers.
You must show all your algebraic expansion and simplification clearly.

.....

(Total for Question is 5 marks)

Worked Answers

I Do

Translating by vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ means moving 4 units right and 3 units down.

Moving 4 right replaces x with $(x - 4)$. Moving 3 down subtracts 3 from the whole function.
$\mathbf{g(x) = f(x - 4) - 3}$.

A stretch parallel to the x-axis with scale factor 2 replaces x with $\frac{1}{2}x$ inside the function.
$\mathbf{h(x) = f(\frac{1}{2}x)}$ or $\mathbf{f(0.5x)}$.

We Do

Translating by vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ means 2 units left and 5 units up.

Moving 2 left replaces x with $(x + 2)$. Moving 5 up adds 5.
$\mathbf{g(x) = f(x + 2) + 5}$.

A stretch parallel to the y-axis with scale factor 3 multiplies the whole function by 3.
$\mathbf{h(x) = 3f(x)}$.

You Do 1

Translating by vector $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ means no horizontal change, and 4 units down.

$\mathbf{g(x) = f(x) - 4}$.

A stretch parallel to the x-axis with scale factor $\frac{1}{3}$ replaces x with $3x$.
$\mathbf{h(x) = f(3x)}$.

You Do 2

Translating by vector $\begin{pmatrix}7\\1\end{pmatrix}$ means 7 units right and 1 unit up.

$\mathbf{g(x) = f(x - 7) + 1}$.

A reflection in the x-axis makes all y-values negative.

$\mathbf{h(x) = -f(x)}$.

You Do 3

A reflection in the y-axis replaces x with $-x$.

$\mathbf{g(x) = f(-x)}$.

A stretch parallel to the y-axis with scale factor 0.5 multiplies the whole function by 0.5.

$\mathbf{h(x) = 0.5f(x)}$ or $\mathbf{\frac{1}{2}f(x)}$.

Challenge 1

The function is $y = -f(x + 2) + 5$.

The $+2$ inside the bracket represents a horizontal translation of 2 units to the left.

The negative sign outside the function represents a reflection in the x-axis.

The $+5$ at the end represents a vertical translation of 5 units upwards.

Sequence of transformations:

1. A translation by the vector $\begin{pmatrix}-2\\0\end{pmatrix}$.
2. A reflection in the x-axis.
3. A translation by the vector $\begin{pmatrix}0\\5\end{pmatrix}$.

(Note: Step 2 must occur before Step 3. Alternatively, a single translation by $\begin{pmatrix}-2\\5\end{pmatrix}$ followed by a reflection in the line $y = 5$ is also valid).

Challenge 2

Original minimum turning point is $(x, y) = (3, -4)$.
The transformation is $y = 2f(x - 1) + 3$.
Inside the bracket, $(x - 1)$ means a translation of 1 unit right.
New x-coordinate $= 3 + 1 = 4$.
Outside the bracket, multiplying by 2 and adding 3 means we apply $2y+3$ to the y-coordinates.
New y-coordinate $= 2(-4) + 3 = -8 + 3 = -5$.
The new minimum turning point is $(\mathbf{4}, \mathbf{-5})$.

Extreme Challenge

Translating by the vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ means a translation of 3 units left and 2 units up.
This transformation applies as $g(x) = f(x + 3) + 2$.
We know $f(x) = x^2 - 4x + 5$.
Substitute $(x + 3)$ in place of x:
$f(x + 3) = (x + 3)^2 - 4(x + 3) + 5$
Expand the bracket $(x + 3)^2$:
$(x + 3)(x + 3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$
Expand the second part:
$-4(x + 3) = -4x - 12$
Combine to find $f(x + 3)$:
$f(x + 3) = x^2 + 6x + 9 - 4x - 12 + 5$
$f(x + 3) = x^2 + 2x + 2$
Now add the $+2$ for the vertical translation:
$g(x) = f(x + 3) + 2 = (x^2 + 2x + 2) + 2$
$\mathbf{g(x) = x^2 + 2x + 4}$